Wholesale Price Discrimination and Recommended Retail Prices*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper shows that legislation which requires recommended retail prices to attract some positive share of sales may act as a commitment device and enable a monopolist manufacturer to efficiently engage in wholesale price discrimination. By creating asymmetries between retailers, where ex-ante there are none, a manufacturer indirectly screens searching consumers. Under wholesale price discrimination, endogenously created low cost retailers sell to a disproportionate share of low search cost consumers, which gives these retailers stronger incentives to compete, resulting in lower retail margins. Moreover, even endogenously created high cost retailers have an incentive to compete more severely given that their competitors have lower prices. We show that under such a discriminatory pricing scheme the average wholesale and retail prices increase, leading to higher manufacturer profits, but lower consumer welfare (and retail profits). We also show that without legislation requiring positive sales at the recommended retail prices, wholesale price discrimination is not possible as the manufacturer would be better off "secretly" giving all retailers the lower wholesale prices. Thus, the paper shows that consumers are better off (and total surplus would be higher) without the legislation aiming to protect them.

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1 Introduction

Recommended retail prices are non-binding recommendations of manufacturers at which prices retailers should sell their product. As retailers are free to deviate from the recommendation, an important question is whether these recommendations affect market behaviour and if so how. Competition authorities scrutinize these recommended retail prices and are worried that competition is negatively affected. In the United States, for example, the Code of Federal Regulations used by the Federal Trade Commission states that “to the extent that list or suggested retail prices do not in fact correspond to prices at which a substantial number of sales of the article in question are made, the advertisement of a reduction may mislead the consumer”. The Code of Federal Regulations rightfully observes that a recommended retail price may also be addressed to consumers (and not only to retailers) and may affect consumers’ purchasing behaviour. For example, consumers may be more inclined to buy at a retailer (instead of continuing to search) if they are informed that the retailer sells at a price below the recommended retail price and this may provide retailers with more market power (up to the recommended retail price) vis-a-vis the consumers that visit their store. In this paper, we argue that it is not in the interest of consumers (or total surplus) to require that a substantial number of sales of the article in question is sold at at the recommended retail price. Despite its intention to protect consumers, the Code of Federal Regulations may actually adversely affect consumers.

To make this point, we consider a market environment where a monopoly manufacturer sets (potentially different) wholesale prices to multiple retailers, while retailers set a retail price given the wholesale price they observe. Retailers sell to final consumers who have heterogeneous search cost and learn about retail prices through sequential search. In this environment, we show that the restrictions imposed by the Code of Federal Regulations effectively provides the possibility for the manufacturer to engage in wholesale price discrimination, making consumers and retailers worse off.

Wholesale price discrimination provides the manufacturer with an opportunity to charge a low wholesale price to some retailers and a high wholesale price to others. As a result (given retailers’ optimal reaction to these asymmetric wholesale prices), there will be low and high retail prices in the downstream market. As consumers do not know which retailers have lower prices, their initial search is random. However, depending on their search cost, consumers will follow different search paths after their first search. Consumers with higher search costs will buy immediately at the first retailer they visit, independent of whether they are at a retailer with a low or a high retail price, since it is too costly for them to continue searching. Low search cost consumers, on the other hand, will only stop searching once they have found the lower retail price. Thus, wholesale price discrimination is a mechanism to (indirectly) screen searching consumers: observing a high retail price low search cost consumers continue to search, while others will buy. As a consequence,
retailers do not face the same composition of search costs among their consumers with low cost retailers’ demand consisting of a relatively larger share of low search cost consumers. As low search cost consumers are more price sensitive, they will induce more competition between low cost retailers. In addition, because of the increased competition between the low cost retailers, consumers with higher search cost may also find it attractive to continue searching forcing the high cost retailer also to lower its margins. Thus, both low and high cost retailers may have lower margins under wholesale price discrimination. As lower retail margins, ceteris paribus increase manufacturer profit, the manufacturer may well consider to engage in wholesale price discrimination to increase profits.

In the absence of the restrictions imposed by the Code of Federal Regulations, wholesale price discrimination cannot, however, be sustained as an equilibrium outcome. The reason is as follows. In the absence of the restrictions, wholesale price discrimination can only be an equilibrium when the manufacturer makes identical profits over all retailers (those that received a low and a high wholesale price). If not, the manufacturer may secretly deviate and charge the same wholesale price to all retailers. However, this equal profit condition cannot be satisfied together with the first-order conditions that the low and high wholesale price have to satisfy. Thus, in the absence of the restrictions imposed by the Code of Federal Regulations, a manufacturer will sell at a uniform price to all retailers. In turn, all retailers will set the same retail price in the downstream market. Foreseeing this, consumers will randomly search one downstream retailer and buy there.

The restrictions imposed by the Code of Federal Regulations effectively provides the manufacturer with a commitment device as it may announce the price at which the high cost retailer sells her product as the recommended retail price. Given the announcement, she should sell at least some products at this price and is not free to deviate and sell to all retailers at the wholesale price generating more profits. We show that eliminating this possible deviation, wholesale price discrimination can be sustained as an equilibrium outcome and that the average wholesale and retail prices increase, increasing manufacturer profits, but decreasing retailers’ profits and consumer welfare.

In summary, our paper suggests that recommended retail prices are ineffective if they are not accompanied by a restriction such as the one imposed by the Code of Federal Regulations. However, often in many markets some form of regulation is in place, and the use of recommended retail prices should not be seen in the light of this regulation. A recommended retail price is a commitment device of the manufacturer that allows him to price discriminate. Some retailers follow the recommended retail price as this is simply their optimal price given their individual wholesale price. Other retailers sell at a price below the recommended retail price as they receive lower wholesale prices. Thus, the observation that recommendations often do not bind in practice as most products sell at a price below the recommended retail price follows naturally from our framework.

There are several branches of the literature to which this paper contributes. First, there
is a recent interest of papers explaining how non-binding recommended retail prices may affect market behavior.\footnote{Two empirical papers (Faber and Janssen (2008) and De los Santos et. al. (2016)) show that recommended retail prices do affect market behavior.} Buehler and Gärtner (2013) and Lubensky \cite{9} use a framework where recommended retail prices are used by the manufacturer to signal production cost. Buehler and Gärtner (2013) see recommended retail prices as communication devices between a manufacturer and her retailers and see recommended retail prices as part of a relational contract enabling the manufacturer and retailer to maximize joint surplus in a indefinitely repeated setting. Lubensky \cite{9} is closer in spirit to our model as he shows that a manufacturer can use recommended retail prices to signal his production cost to searching consumers. As both consumers and the manufacturer prefer more search when the manufacturer production cost is low and less search when it is high, the manufacturer’s recommendation informs consumers via cheap talk of its cost. In contrast to these papers, uncertainty concerning manufacturer cost does not play a role in our setting. This implies that if the manufacturer sets different wholesale prices, he should either be indifferent between them, or there should be some rule allowing him to use recommended retail prices as a commitment device. The use of recommended retail prices should not be seen independent of the regulations that governs them.

Second, the idea that a monopolist may want to sell at different prices to discriminate between consumers with different search cost is not new. In fact, Salop (1977) argues that a monopolist may want to sell at higher prices to less price-sensitive consumers with higher search cost, while selling at lower prices to consumers with lower search cost. His argument, however, critically depends on the assumption that the monopolist is committed to charging prices according to a price distribution and that any deviation from this distribution is observed by consumers and consumers will react by changing their search strategy. From a formal game theoretic point of view it is difficult to see how consumers may observe a price distribution, while maintaining the assumption underlying the search cost literature that the consumer does not know the prices the firm sets. Without this commitment, Salop’s argument breaks down, however, as the monopolist will have an incentive to secretly increase the prices in the lower part of the price distribution. Our paper shows that without commitment, screening consumers with different search costs can be effective in a vertical relations model where the manufacturer imposes the screening contract to retailers, while consumers search for low retail prices.

Third, there is a literature on price discrimination in intermediate goods markets. The seminal papers in this literature, Katz \cite{8}, DeGraba \cite{3} and Yoshida \cite{14}, have built arguments in favour of banning price discrimination. The basic starting point in these papers is that downstream firms differ in their efficiency levels. An unconstrained monopolist manufacturer may then choose to charge higher wholesale prices to more efficient firms. Uniform pricing constraints the monopoly power of the manufacturer increasing total
surplus. Inderst and Valleti [6] and O’Brien [11] show that a ban on discrimination may have the opposite effect if the assumption of an unconstrained manufacturer is relaxed. In our paper, in contrast, retailers are ex ante symmetric and would have the same cost if the manufacturer would engage in uniform pricing. The novelty of our paper is that in many markets consumers must engage in costly search to get to know market prices. By taking into consideration information frictions regarding retail prices and retailers’ marginal costs, a manufacturer may purposefully create asymmetries between retailers by engaging in wholesale price discrimination (if it can be supported as an equilibrium outcome).

Fourth, there is a small literature on vertically related industries with consumer search (Janssen and Shelegia [7], Garcia, Honda, and Janssen [5], Garcia and Janssen [4] and Asker and Bar-Isaac [1]). Janssen and Shelegia [7] show that markets can be quite inefficient if consumers search sequentially while not observing the wholesale arrangement between the manufacturer and retailers. Garcia, Honda, and Janssen [5] extend that argument to wholesale markets where retailers search sequentially among different manufacturers. Both these papers assume that manufacturers treat retailers symmetrically and do not engage in wholesale price discrimination (although the latter paper allows for manufacturers to randomize their decision to choose wholesale prices). Garcia and Janssen [4] allows for wholesale price discrimination, but mainly focuses on how a manufacturer can correlate his wholesale prices to increase profits. By contrast, we focus on the competitive impact of wholesale price discrimination by changing the search cost composition of different retailers. In contrast to Garcia and Janssen [4], low cost retailers will not have monopoly power in our context as the manufacturer chooses to have at least two retailers getting the low wholesale price, maintaining competitive pressure between them. Finally, Asker and Bar-Isaac [1] study the impact of minimum advertised prices (MAPs). They see different potential roles for MAPs with price discrimination as one of them. Their model of price discrimination is very different from ours, however. They study a market where consumers have different valuations and the consumer search cost distribution is "Varian (1980) like", where some consumers compare all prices and others always buy at the first store they visit (as their search cost is prohibitively high). The rationale for wholesale price discrimination in their paper is therefore close to the traditional role for price discrimination in extracting surplus from consumers with different valuations. In contrast, in our model consumers have identical valuations and wholesale price discrimination is a way to screen consumers with different search cost. By imposing wholesale price discrimination the manufacturer endogenously determines how many consumers search beyond the first firm. We therefore have a pure informational story of

\[2\] This is also the reason why we prefer to speak of wholesale markets and not of intermediate goods, or input markets. Wholesale markets stress that the only difference between retailers may be caused by manufacturers selling at different prices.
price discrimination.

Finally, while most papers in the search literature assume at most two different levels of search cost (see, e.g., Stahl [12]), there do exist some papers that consider more general forms of heterogeneity in consumers’ search costs, such as Stahl [13], Chen and Zhang [2] and Moraga-González, Sándor, and Wildenbeest [10]. In contrast to these papers, however, we focus on vertical related industry structures. In line with this literature, as the measure of consumers with zero search cost is zero in our model, there exists a continuum of equilibria. In all of the different settings we consider, we focus on the equilibrium that maximizes total surplus (as this is also the equilibrium where the manufacturer makes most profit).

The remainder of this paper is organized as follows. In the next section, we present the details of the model we consider and the equilibrium concept we use. The benchmark model with uniform wholesale prices is discussed in Section 3 where we also discuss the issues of multiplicity of equilibria. Section 4 presents the equilibrium analysis allowing for wholesale price discrimination. This Section also discusses in more detail why wholesale price discrimination only works if the manufacturer is allowed to impose MAPs. The comparison with the benchmark model for the linear demand case is also given there. Section 5 analyses the implications of imposing that some sales take place at the list price, while section 6 concludes.

2 The Model

We focus on a vertically related industry with a monopolist manufacturer in the upstream market supplying a homogeneous good to \( N \geq 3 \) retailers.\(^3\) The manufacturer’s production costs are normalized to zero. The wholesale price charged by the manufacturer will be denoted by \( w \). In principle the manufacturer can charge a different wholesale price to every retailer so that formally the manufacturer’s strategy is a tuple \((w_1, w_2, ..., w_N)\). We will focus on two types of equilibria: (i) in a uniform pricing equilibrium the manufacturer chooses \( w_i = w^* \), whereas in an equilibrium with price discrimination the manufacturer chooses two prices \( w^*_L \) and \( w^*_H \), with \( w^*_L < w^*_H \), and charges some retailers the low and others the high wholesale price. Retailers take their wholesale price as given and do not have other cost except for the wholesale price paid to the manufacturer for each unit they sell. Observing only their own wholesale price retailers compete in prices and choose their retail strategy \( p(w) \).

There is a unit mass of consumers each demanding \( D(p) \) units of the good if they buy at price \( p \). There exists a \( \bar{p} \) such that \( D(p) = 0 \) for all \( p \geq \bar{p} \) and the demand function is

\(^3\)To study the effects of wholesale price discrimination, it is important there are at least three retailers so that there are at least two retailers that get the lowest wholesale price and there is still some competition among these retailers.
downward sloping whenever demand is strictly positive, i.e., $D'(p) < 0$ for all $0 \leq p < \bar{p}$. Consumers differ in their search cost, which are denoted by $s$, and we assume these are uniformly distributed on the interval $[0, \bar{s}]$. In order to observe prices consumers have to engage in costly sequential search with perfect recall. For most part of the analysis, it does not matter whether or not the first search is costly, but not to complicate the analysis too much we proceed assuming the first search is for free so that we do not have to consider the participation constraint of consumers (which for small enough $\bar{s}$ will always be satisfied). As consumers are not informed about prices before they search, an equal share of consumers visit each retailer at the first search.

The timing of play is as follows. First, the manufacturer sets wholesale prices to all firms. Each retailer observes the wholesale price that the manufacturer offers them. they do not observe the wholesale prices offered to other retailers. Like Janssen and Shelegia (2015) it is important that consumers do not observe wholesale prices. Second, given their individual $w_i$, each retailer $i$ sets their retail price $p_i$, where $i = 1, ..., N$. Finally, consumers sequentially search for retail prices.

Perfect Bayesian Equilibrium (PBE) is used as a solution concept and we focus on symmetric-pure strategy equilibria. On the equilibrium path, retailers’ and consumers’ beliefs are updated using Bayes’ rule, however it is important to consider out-of-equilibrium beliefs carefully. Let us first consider an equilibrium with uniform wholesale pricing, denoted by $(w^*, p^*(w))$, and consider a consumer who observes a price $p$ different from $p^*(w^*)$. To determine how a retailer optimally reacts to a wholesale price $w^*$ it is important to specify how a consumer reacts to a deviation from $p^*(w^*)$. This in turn depends on consumer beliefs about prices they believe they will encounter if they continue to search. For example, if consumers would have symmetric beliefs, they would believe that other retailers would set the same price if they observe a price $p \neq p^*(w^*)$ and in this case, they will decide not to continue to search. Symmetric beliefs would give full monopoly power to retailers, independent of the search cost distribution. If, on the other hand, consumers would have passive beliefs, they would believe that other retailers continue to set $p^*(w^*)$ if they observe a price $p \neq p^*(w^*)$ and in this case, the consumers with low enough search cost will continue to search if the price they observe is such that $p > p^*(w^*)$. As consumer search plays an important role in our analysis, we will adopt passive beliefs when consumers observe an out-of-equilibrium price. For retailers, the issue is a little more subtle, but given the passive beliefs of consumers it seems most natural to also impose passive beliefs on retailers.$^4$

Thus, we define a uniform pricing equilibrium as follows.

$^4$One may argue that if retailers know, or believe, that the manufacturer has a uniform pricing policy this also should apply out-of-equilibrium, i.e., if a retailer observes a wholesale price $w \neq w^*$, then he should also expect other retailers to have a unit cost of $w$. This alternative assumption on out-of-equilibrium beliefs significantly complicates the analysis as retailers would not react with a pure strategy any more. To see this note first that if a pure strategy reaction $p^*(w)$ would exist it should be different from $p^*(w^*)$ as otherwise the manufacturer would have an incentive to deviate to higher prices. But
Definition 1 A uniform pricing equilibrium is defined by a tuple \((w^*, p^*(w))\) and an optimal sequential search strategy for all consumers such that (i) the manufacturer maximizes profits given \(p^*(w)\) and consumers’ optimal search strategy, (ii) retailers maximize their retail profits given the wholesale price they observe, their beliefs about the wholesale prices observed by other retailers and consumers’ optimal search strategy and (iii) consumers’ sequential search strategy is optimal given \((w^*, p^*(w))\) and their beliefs about retail prices not yet observed. Beliefs are updated using Bayes’ rule whenever possible. Off-the-equilibrium path, beliefs are passive, i.e.,

- Retailers always believe that their competitors received a wholesale price \(w^*\) independent of the wholesale price they observed themselves;
- Consumers believe that retailers that are not searched yet have set a retail price \(p^*(w^*)\) independent of the retail price(s) they already observed.

Next consider an equilibrium with wholesale price discrimination, which is denoted by \(((w^*_L, w^*_H), p^*(w))\). Again, we will impose a form of passive beliefs, but now have to be more specific. First, the equilibrium should specify how many retailers observe the low and how many retailers observe the high wholesale price. Suppose on the equilibrium path there are \(1 \leq m^* \leq N - 1\) retailers that received \(w_L\) and the remaining \(N - m^*\) received \(w_H\). A consumer observes an on-the-equilibrium path price of \(p^*(w^*_L)\) believes that if he continues to search, there is a probability of \(\frac{N-m^*}{N-1}\), respectively \(\frac{m^*-1}{N-1}\), he will observe a price of \(p^*(w^*_H)\), respectively \(p^*(w^*_L)\) on his next search. However, if the consumer observes an on-the-equilibrium path price of \(p^*(w^*_H)\) he believes that if he continues to search, there is a probability of \(\frac{N-m^*-1}{N-1}\), respectively \(\frac{m^*-1}{N-1}\), he will observe a price of \(p^*(w^*_H)\), respectively \(p^*(w^*_L)\) on his next search. That is, even on the-equilibrium path the beliefs about retail prices on the next search depend on which retail price is observed.

Consider then a consumer who observes a price \(p\) slightly larger than \(p^*(w^*_H)\). Even if he has passive beliefs, he has to have a belief whether it was a high or a low cost retailer that has deviated. We will argue that an equilibrium requires that at prices \(p\) in the neighbourhood of \(p^*(w^*_H)\) the consumer believes it is a high-cost retailer that has deviated. The reason is as follows. Suppose that the consumer randomly attributes the deviation price, or that he attributes it to a low cost retailer. In that case, after observing a price \(p > p^*(w^*_H)\) the consumer would become more pessimistic about finding lower prices on his next search than after observing the equilibrium price \(p^*(w^*_H)\). More consumer would then decide not to continue searching if they observe a deviation price \(p\) in the neighbourhood of \(p^*(w^*_H)\) than after observing \(p^*(w^*_H)\), but this would make it profitable to deviate for a high cost retailer. Thus, to have an equilibrium it is necessary retailers know that if they set a price different from \(p^*(w^*)\) some consumers would continue to search and observe at least two different prices. To attract these consumers it would then be better to undercut the price of the competitors. Thus, a pure retail strategy does not exists if retailers have symmetric beliefs.
that consumers attribute deviation prices in the neighbourhood of \( p^*(w^*_H) \) to a retailer that was supposed to have a high cost. If a consumer observes other out-of-equilibrium prices, we are more free to specify which retailer the consumer blames for such a price. Therefore, in the equilibrium definition below we do not restrict these beliefs further than necessary. Not to have our results be driven by out-of-equilibrium beliefs that favour retail competition, in the main part of the analysis we will say that consumers attribute deviations to a low cost retailer if the deviation price \( p \) in the neighbourhood of \( p^*(w^*_L) \) so that beliefs are continuous in a neighbourhood of both equilibrium prices. However, we will also perform an analysis to investigate the robustness of our results. In particular, we also investigate what will happen when consumers blame high cost retailers for all deviations where \( p > w^*_H \). For consistency reasons, we always invoke similar beliefs for retailers.

Thus, we define an equilibrium with wholesale price discrimination as follows.

**Definition 2** An equilibrium with wholesale price discrimination is defined by a tuple \(((w^*_L, w^*_H), p^*(w), m^*)\), with \( w^*_L < w^*_H \), and an optimal sequential search strategy for all consumers such that (i) the manufacturer maximizes profits given \( p^*(w) \) and consumers’ optimal search strategy, (ii) retailers maximize their retail profits given the wholesale price they observe, their beliefs about the wholesale prices observed by other retailers and consumers’ optimal search strategy and (iii) consumers’ sequential search strategy is optimal given \((w^*, p^*(w))\) and their beliefs about retail prices not yet observed. Beliefs are updated using Bayes’ rule whenever possible. Off-the-equilibrium path are passive and satisfy at least the following restrictions:

- A retailer observing a wholesale price \( w \) in the neighbourhood of \( w^*_H \) believes that \( m \) competitors receive a wholesale price of \( w^*_L \), while the remaining \( N - m - 1 \) competitors receive a wholesale price of \( w^*_H \);

- If consumers observe a retail price \( p \) in the neighbourhood of \( p^*(w^*_H) \) they believe that a high cost retailer is responsible for setting this price.

3 Uniform pricing

We begin by analysing the benchmark case with uniform pricing, in which the manufacturer charges the same wholesale price to all retailers. The case where the monopolist manufacturer charges different prices to retailers, is analysed in the next section.

First, we characterize the behaviour of consumers and retailers. Let \( p^*(w^*) \) denote the equilibrium price charged by all retailers (and the retail price consumer expect). All consumers become active and search for prices if either the first search is for free or is their expected surplus if they are active, given by \( \int_{p^1}^{1} D(p)dp \), is larger than the maximal search cost \( \bar{s} \).
Fig. 3 Share of consumers that buy at deviant retailer

If a consumer buys at a deviation price $\tilde{p} > p^*$, he gets a consumer surplus of $\int_{p^*(w^*)}^{\tilde{p}} D(p) \, dp$. Thus, using passive beliefs as defined in the previous section, a consumer with search cost $s$ continues to search for the equilibrium price $p^*(w^*)$, if

$$s < \int_{p^*(w^*)}^{1} D(p) \, dp - \int_{p^*(w^*)}^{\tilde{p}} D(p) \, dp.$$

Thus, a retailer who deviates to a price $\tilde{p} > p^*(w^*)$ gets a fraction $s\left(\int_{p^*(w^*)}^{1} D(p) \, dp - \int_{p^*(w^*)}^{\tilde{p}} D(p) \, dp\right)$ of consumers who initially come to buy from him. Therefore the deviating retailer’s profit in a uniform pricing equilibrium equals:

$$\pi_r(\tilde{p}, p^*) = \frac{1}{N} \left(\int_{p^*(w^*)}^{1} D(p) \, dp - \int_{p^*(w^*)}^{\tilde{p}} D(p) \, dp\right)\tilde{p} - w.$$

Note that at the retail monopoly price, denoted by $p^M(w)$, we have $D'(p^M(w))(p^M(w) - w) + D^2(p^M(w)) = 0$. A retailer will never set a price larger than the retail monopoly price as he can always guarantee himself the retail monopoly profits (by lowering his price). Even if he does not attract additional consumers by doing so, he would make more profits over consumers that anyway will visit him. Thus, we must have that in equilibrium $D'(p^*(w^*)p^*(w^*) + D(p^*) \geq 0$.

Maximizing retail profits using the equilibrium condition $\tilde{p}(w^*) = p^*(w^*)$, yields that $p^* \leq p^M(w^*)$ and

$$-\frac{D^2(p^*)(p^* - w^*)}{\bar{s}} + D'(p^*)(p^* - w^*) + D(p^*) \leq 0. \quad (1)$$

Note that the first-order condition has to be satisfied with a weak inequality as firms will never have an incentive to lower their price as long as $p^* \leq p^M(w^*)$ given that consumer search for lower prices and do not observe these prices until at the retailer in question. Thus, retailers do not attract more consumers by lowering their prices.

Also, the equilibrium retail price is independent of the number of active retailers. However, this does not imply that if retailers and consumers would expect that some retailers are foreclosed by the manufacturer (for example by receiving such a high wholesale price

\footnote{Note that here we assume that consumers have passive beliefs and blame an individual retailer for the out-of-equilibrium price. If consumers would blame the manufacturer and believe that all retailers set the same price (in reaction to a uniform deviation of the manufacturer), then retailers have monopoly power and the double marginalization outcome would prevail.}
that they cannot effectively compete), while all the remaining retailers receive identical wholesale offers, retailers would behave in exactly the same way as in the case when all $N$ retailers would receive the same wholesale price. The reason is that in the above analysis, it is taken for granted that if a consumer continues to search he will always find the equilibrium price on the next search. This will not be the case under foreclosure, however, as in that case the chance of finding a low retail price will be smaller and consumers will be more reluctant to search. This gives retailers more market power. Thus, a manufacturer will not want to foreclose retailers from the market. Wholesale price discrimination is, as we will see in the next Section, more subtle than foreclosure.

To determine the wholesale equilibrium price under uniform pricing, we first should note that in an equilibrium in the vertical model it can never be the case that (1) holds with strict inequality. The reason is that in that case the manufacturer could increase profits by increasing her wholesale price. This will always be profitable as retailers will not adjust their retail price and therefore the manufacturer demand will not be affected.

In addition, in an equilibrium it should not be optimal for a manufacturer to deviate to one retailer and give him $w$ (keeping the other retailers at $w^*$). If the manufacturer would deviate in this way, his profits are given by

$$\pi(w^*, w') = \left( \frac{N}{N} - 1 + \frac{1}{N} \int \hat{p}(w) \, D(p) \, dp \right) w^* D(p^*(w^*)) + \frac{1}{N} \left( 1 - \frac{1}{N} \int \hat{p}(w) \, D(p) \, dp \right) w D(\hat{p}(w)).$$

This expression is easily understood. Of the consumers who encounter a price of $\hat{p}(w)$ at their first search (which is a fraction $1/N$ of them) continue to search for the equilibrium retail price if their search cost is smaller than $\int \hat{p}(w) \, D(p) \, dp$. The consumers with a higher search cost, which is a fraction $1 - \frac{1}{N} \int \hat{p}(w) \, D(p) \, dp$, will buy at the deviation price $\hat{p}(w)$. All other consumers buy at the equilibrium price $p^*(w^*)$.

A uniform pricing equilibrium requires that $\frac{\partial \pi}{\partial w'}$ evaluated at $w' = w^*$ is smaller than or equal to 0:

$$\frac{\partial \pi}{\partial w'} = \frac{1}{N} \int \hat{p}(w') \, D(p) \, dp \left( w^* D(p^*_1(w^*)) - w' D(\hat{p}(w')) \right) + \frac{1}{N} \left( 1 - \frac{1}{N} \int \hat{p}(w') \, D(p) \, dp \right) \left( w' D'(\hat{p}) \frac{\partial \hat{p}}{\partial w'} + D(\hat{p}) \right) \leq 0,$$

which reduces to

$$w^* D'(\hat{p}(w^*)) \frac{\partial \hat{p}(w^*)}{\partial w'} + D(\hat{p}(w^*)) \leq 0. \quad (2)$$

As with the retailer’s maximization problem, the manufacturer does not have an incentive to lower his wholesale price. As retailers will not follow suit and keep their price at the equilibrium level as long as $p^* < p^M(w^*)$, the manufacturer also does not have an incentive to lower his price from the equilibrium level (whatever that level is). The only requirement we have to impose is that the manufacturer does not want to increase his wholesale price and this is what (2) requires.
To finalize the description of an equilibrium, we still have to evaluate how \( \tilde{p} \) changes with a change in \( w \). For this we need to determine the best response function of retailers to non-equilibrium wholesale prices. Given the retailers’ profit function, we get that an individual retailer will react to deviations in \( w \) by setting \( \tilde{p} \) such that

\[
- \frac{D^2(\tilde{p})}{\bar{s}}(\tilde{p} - w) + \left( \bar{s} - \left( \int_{\tilde{p}}^{p^*} D(p) dp - \int_{\tilde{p}}^{p^*} D(p) dp \right) \right) \left( D'(\tilde{p})(\tilde{p} - w) + D(\tilde{p}) \right) = 0. \tag{3}
\]

Thus, the retailer’s best response to any \( w \) depends on \( w \) itself as well as on the equilibrium price \( p^*(w^*) \) that is expected by consumers. Observe that in this equilibrium the retailer’s reaction is smaller than the retail monopoly price due to the fact that low search cost consumers continue to search if a retailer would charge at this price. In the proof of the next Proposition we show that if we evaluate this reaction at the equilibrium values we get

\[
\frac{\partial \tilde{p}(w^*)}{\partial w} = \frac{\bar{s}D'(p^*) - D(p^*)}{3D'(p^*)(p^* - w^*) - 2D(p^*) + \bar{s}\left( D''(p^*)(p^* - w^*) + 2D'(p^*) \right)} \tag{4}
\]

where we use \( p^* \) as a short-hand notation for \( p^*(w^*) \).

We then have the following result.

**Proposition 3** The uniform pricing equilibrium that maximizes total surplus is given by (1) and (2), both holding with equality, where \( \frac{\partial \tilde{p}(w^*)}{\partial w} \) is given by (4).

From (1) it can be seen that as \( \bar{s} \to 0 \), \( p^*(w^*) \to w^* \). This is quite intuitive: when the consumer search cost become arbitrarily small, retailers do not have any market power and their retail margins should become arbitrarily small as well. What is perhaps more surprising is that when \( \bar{s} \to 0 \) and \( p^* \to w^* \) we can solve (2) for \( w^* \). It turns out that when \( \bar{s} \to 0 \) the expression for \( \frac{\partial \tilde{p}(w^*)}{\partial w} \) reduces to \( \frac{1}{2} \) so that the wholesale price is significantly larger than that of an integrated monopolist (in which case \( \frac{\partial \tilde{p}(w^*)}{\partial w} \) equals 1). The next Proposition states the result.

**Proposition 4** When \( \bar{s} \to 0 \) the uniform pricing equilibrium converges to \( p^* = w^* \), where \( w^* \) solves \( \frac{1}{2}w^*D'(w^*) + D(w^*) = 0 \).

This result is akin to the limiting result in Janssen and Shelegia (2015) where they show in the context of a Stahl (1989) type model at the retail level that as \( \bar{s} \to 0 \), wholesale and retail price converge to a price \( w^* \) that solves \( \lambda w^* D'(w^*) + D(w^*) = 0 \), where \( \lambda \) is the fraction of shoppers in their model. Their equilibrium only exists, however for \( 1 > \lambda > \lambda^* \approx 0.47 \) so that for many parameter values their limit prices are lower than in our model with truly heterogeneous search cost.
3.1 Linear Demand

For linear demand $D(p) = 1 - p$, the above expressions can be simplified considerably. The retail equilibrium price in the welfare maximizing equilibrium should satisfy:

$$-(1-p^*)^2(p^*-w^*) + 1 - (2p^*-w^*) = 0. \quad (5)$$

As for linear demand

$$\frac{\partial \hat{p}(w^*)}{\partial w} = \frac{(1-p^*) + \frac{\tau}{1-p^*}}{2(1-p^*) - 3(p^*-w^*) + \frac{2\tau}{1-p^*}},$$

the manufacturer’s equilibrium wholesale price should satisfy

$$1 - p^* - w^* \frac{(1-p^*) + \frac{\tau}{1-p^*}}{2(1 + w^* - 2p^*) - (p^* - w^*) + \frac{2\tau}{1-p^*}} = 0. \quad (6)$$

The equilibrium under uniform pricing should satisfy (6) and (5). In the case of linear demand we can explicitly solve for $w^*$ when $\tau \to 0$ and $p^* \to w^*$ as we get that in the limit (6) reduces to $1 - w^* - w^*/2 = 0$ so that $w^* \to 2/3$. We can also derive that when $\tau$ is small, both $p^*$ and $w^*$ are decreasing in $\tau$ with $w^*$ decreasing much faster than and $p^*$. This is the content of the next Proposition.

**Proposition 5** When $\tau$ is small enough, under uniform pricing with linear demand, both $p^*$ and $w^*$ are decreasing in $\tau$ with $p^*$, while expected consumer surplus is increasing in $\tau$. In a neighbourhood of $\tau = 0$, we have that expected consumer surplus converges to $\frac{1}{15}$, $\frac{dp^*}{d\tau} = -2$, $\frac{dw^*}{d\tau} = -5$ and $\frac{dESC}{d\tau} = \frac{2}{3}$.

For larger values of $\tau$ we can solve (6) and (5) numerically. Figure 3.1. shows how the equilibrium retail and wholesale price change for different values of $\tau$. It also confirms that when $\tau \to 0$ retail margins are very small and $w^* \to 2/3$. Moreover, initially, for small values of $\tau$ the figure also confirms that both $p^*$ and $w^*$ are decreasing in $\tau$.

![Figure 3.1. Uniform retail and wholesale prices for different values of $\tau$](image)
4 Wholesale Price Discrimination

In this section we focus on the case in which the monopolist manufacturer can set different wholesale prices to different retailers and consider the manufacturer charges a low price $w^*_L$ to $m^*$ retailers and a higher price $w^*_H$ to the remaining $N - m^*$ retailers. To keep a competitive constraint on the retailers that receive the low wholesale price, it should be that $m^* \geq 2$ and this is why, for wholesale price discrimination to make sense we require that $N \geq 3$. When there is no confusion we also use the notation $p(w_i)$, or simply $p_i$, $i = L, H$, to denote price reactions (or prices) of retailers who have received a low or a high wholesale price. Abusing notation a little, we also this notation when a retailer has received a wholesale price that is slightly larger than the equilibrium prices that are expected; see Section 2 for more detail on how retailers form beliefs.

If retailers react to wholesale price discrimination by choosing $p^*_L$ and $p^*_H$, with $p^*_L < p^*_H$, then we can define a critical search cost value $\hat{s}$ as:

$$\left( \frac{m^*}{N-1} + \frac{N - m^* - 1}{N-2} \frac{m^*}{N-1} + \ldots + \frac{N - m^* - 1}{N-2} \frac{N - m^* - 2}{N-3} \ldots \cdot 1 \right) \hat{s} = \int_{p^*_L}^{p^*_H} D(p)dp$$

This expression can be understood as follows. If the equilibrium involves $m^* - 1$ retailers receiving $w^*_L$ then the LHS of this expression reduces simply to $\hat{s}$. In that case, a consumer that encounters $p^*_H$ on her first search expects all other retailers to have chosen $p^*_L$ and therefore she is indifferent between buying at $p^*_H$ or continuing to search if the equality holds. Consumers with a search cost smaller than $\hat{s}$ will find it worthwhile to continue searching, while for the consumers that have search costs higher than $\hat{s}$, it will not be worthwhile to search for the low retail price. If the equilibrium involves less than $m^* - 1$ retailers receiving $w^*_L$ then the LHS of this expression is more complicated as there is a probability that the consumers will not immediately encounter $p^*_L$ on her next search.

What is true, however, is that if a consumer continues to search after the first observation of $p^*_H$ he will certainly continue to search after observing $p^*_H$ each subsequent time as the chance of observing $p^*_L$ on the next search round becomes higher. Thus, consumers can be divided into two groups. First, consumers with a search cost $s > \hat{s}$, for some $\hat{s}$, to be determined in equilibrium, stop and buy at the first retailer even if they observe the higher price $p^*_H$. Second, consumers with a search cost $s < \hat{s}$, who continue to search if they observe the higher price $p^*_H$. Of course, all consumers stop and buy if they observe the lower price $p^*_L$.

Importantly, as we have argued in the previous Section, it will never be optimal for the manufacturer to induce an equilibrium where $\hat{s} \geq \bar{s}$. If that would be an equilibrium, retailers receiving a high wholesale offer reacting with a retail price $p^*_H$ would be effectively foreclosed from the market and this gives the remaining retailers more market power enabling them to have higher retail margins. Thus, in the remaining of this Section, we only consider equilibria with wholesale price discrimination where $0 < \hat{s} \leq \bar{s}$.
Given the discussion in Section 2 on the out-of-equilibrium beliefs around $p_H^*$ necessary to sustain an equilibrium with wholesale price discrimination, it is clear that we should have that the first-order conditions for the high cost retailers and for the manufacturer at $w_H^*$ should be satisfied with equality. The reason is that in such an equilibrium, a fraction of consumers continues to search, namely those with search cost $s < \hat{s}$ that first visit a high-price retailer. If a high-cost retailer would deviate from the equilibrium price in upward or downward direction (either himself or in reaction to a deviation from the manufacturer) his demand changes continuously. As equilibrium requires that such deviations are not optimal, the two respective first-order conditions should hold with equality. This is not the case, however, for low-cost retailers or for the manufacturer at $w_L^*$ as here we only need to upward deviations in prices are not optimal. This is for the very same reason why in an equilibrium with uniform pricing we only need that the relevant derivatives evaluated at their equilibrium values are non-positive: consumers will only find out about the deviations once they have visited the retailer in question. Downward deviations in retail price therefore do not attract additional demand and this makes such deviations always unprofitable.

The flexibility that arises from the fact that the first-order conditions at the low wholesale and retail price do not have to hold with equality is essential for the investigation of wholesale price discrimination. As the manufacturer can always secretly deviate in the number $m$ of retailers getting the low wholesale price without either retailers consumers noticing it, we have to have that in equilibrium with wholesale price discrimination $w_H^*D(p_H^*) = w_L^*D(p_L^*)$. Generically, it is impossible to satisfy this additional restriction if both the manufacturer and the retailer first-order conditions at $w_L^*$ and $p_L^*$, respectively, have to hold with equality.

For now we perform the analysis for $m^* = N - 1$ leaving the case for general $m$ for later.\footnote{In this case, a consumer who has first visited a high price retailer, expects to visit a low price retail for sure if she decides to continue searching.} If $m^* = N - 1$, then the expression for $\hat{s}$ simplifies to $\hat{s} = \int_{p_H}^{p_H^*} D(p)dp$.

If a high cost retailer deviates to a price $p_H$ slightly larger than $p_H^*$, then he will lose all consumers that have\footnote{Referring to the discussion above on consumers finding it more and more profitable to continue to search if they keep on observing prices $p_H^*$, this expression uses the fact that if a consumer continues to search for price slightly higher than $p_H^*$ he will certainly continue to search if he observes a price of $p_3^*$ in the next search round. If a retailer deviates to even higher prices, he may lose even more consumers as some consumers, making the deviation even more unprofitable.}

$$s < \hat{s} + \int_{p_H}^{1} D(p)dp - \int_{p_H}^{1} D(p)dp.$$  

Therefore, the profit of a retailer who has observed a wholesale price $w_H$ in the neighbourhood of $w_H^*$ and sets a price $p_H$ in the neighbourhood of $p_H^*$ will be:

$$\pi_r^H(p_H, p_H^*; w) = \frac{\bar{s} - \left(\hat{s} + \int_{p_H}^{p_H^*} D(p)dp\right)}{N\bar{s}}D(p_H)(p_H - w_H).$$
Taking the first-order condition with respect to $p_H$ and substituting $p_H = p_H^*$ yields
\[
- \frac{D^2(p_H^*)(p_H^* - w_H)}{\bar{s} - \hat{s}} + \left[ D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right] = 0. \tag{7}
\]

Comparing this FOC with that in (1) of the uniform pricing equilibrium in the previous Section reveals that the main difference is that the numerator of the first term is strictly smaller. As this first term is negative, ceteris paribus this implies that high cost retailers will have lower margins. This is one of the important effects of wholesale price discrimination discussed in the Introduction: as (some) competitors have lower retail prices, it is more attractive for consumers to continue searching if they have visited a high cost retailer imposing a more severe competitive constraint on these retailers. High cost retailers have fewer buying customers and a deviation from the equilibrium price will cause the same absolute number of consumers to leave as in the uniform pricing equilibrium. Therefore, in relative terms, the impact on the retailer of consumers leaving is larger.

Like in the previous Section, (1) reveals that in any equilibrium with wholesale price discrimination if $\bar{s}$ becomes arbitrarily small, it has to be the case that $p_H^*$ becomes arbitrarily close to $w_H^*$. If $\bar{s}$ becomes arbitrarily small, so does $\hat{s}$ and therefore the same argument applies as in the previous Section.

If a low cost retailer deviates to a price $p_L$ with $p_L^* < p_L < p_H^*$, then his profit function is
\[
\pi_r^L(p_L, p_L^*, p_H) = \frac{1}{N} \left[ \frac{\bar{s} - N - 1}{N} \int_{p_L}^{p_L^*} D(p)dp + \frac{1}{N(N - 1)\bar{s}} \int_{p_L}^{p_H^*} D(p)dp \right] D(p_L)(p_L - w_L).
\]

There are two important differences with respect to the uniform pricing case. First, as low cost retailers have a disproportionately large share of low search cost consumers, they are losing relatively more consumers if they would increase their prices. This is reflected in the second term in the square brackets. Second, as consumers do not know the prices of the other retailers and they expect one retailer to have higher prices, they are in principle less inclined to continue searching compared to the uniform pricing case. This is reflected in the $\frac{N - 1}{N}$ term in the first term in the square brackets.

The FOC with respect to $p_L$ gives
\[
0 = \left[ 1 - \frac{N - 1}{N\bar{s}} \int_{p_L}^{p_L^*} D(p)dp + \frac{1}{N(N - 1)\bar{s}} \int_{p_L}^{p_H^*} D(p)dp \right] \left[ D'(p_L)(p_L - w) + D(p_L) \right]
- \left( 1 + \frac{1}{N(N - 1)} \right) \frac{D^2(p_L)(p_L - w)}{\bar{s}},
\]

which evaluated at the equilibrium value yields
\[
- \frac{\left( N - 1 + \frac{1}{N} \right) D^2(p_L^*)(p_L^* - w_L^*)}{(N - 1)\bar{s} + \int_{p_L}^{p_H^*} D(p)dp} + \left[ D'(p_L^*)(p_L^* - w_L^*) + D(p_L^*) \right] = 0. \tag{8}
\]
The most important differences with respect to uniform pricing are that (i) as explained above, the first-order condition should only hold for upward deviations, hence the inequality, and (ii) that the fraction is multiplied by a factor $1 + \frac{1}{N(N-1)} > 1$. Especially, when $N$ is small, this term may create an important difference and illustrates the other important effect of wholesale price discrimination as discussed in the Introduction: even though low search cost consumers may be less inclined to continue to search (as they may not directly find another low cost retailer), the fact that low cost retailers are more frequently visited by low search cost consumers outweighs this effect.

To finalize the equilibrium description under wholesale price discrimination, we write the manufacturer’s profit function for an arbitrary value of $p_H$ and for the case where he deviated to one low cost retailer. We get $\Pi_M =$

$$\frac{1}{N} \left( 1 + \frac{1}{(N-1)s} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \right) \left( \frac{w_H - \frac{1}{N} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \left( \tilde{p}_L(w_L) \right)}{ \frac{N-1}{N^2} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp } \right) w_L D(\tilde{p}_L(w_L))$$

$$+ \frac{N-2}{N} \left( 1 + \frac{\int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \left( \tilde{p}_L(w_L) \right)}{(N-1)s} \right) \frac{\int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \left( \tilde{p}_L(w_L) \right)}{(N-1)(N-2)s} \left( \frac{N-1}{N^2} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \right) w_L^* D(p_L^*(w_L^*))$$

$$+ \frac{1}{N} \left( 1 - \frac{1}{s} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \right) w_H D(p_H(w_H)).$$

This expression can be understood as follows. First, $\frac{1}{N^2} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp$ is the share of consumers that first saw $p_H(w_H)$ and continues to search as they believe that all other firms choose $p_L^*$. Each of the other retailers gets $1/(N - 1)$ of these consumers. Retailers charging $p_L^*$ will sell to these consumers, while a retailer that charges $p_L$ will only get a fraction of these consumers, namely those with relatively higher search cost. Since they still believe that the other retailers charge $p_L^*$, all consumers with a search cost smaller than $\int_{p_L(w_L)}^{p_H(w_H)} D(p) dp$ continue searching for the remaining retailers and buy there. Finally, there is a share of consumers that on their first search observes $p_L$. As the low search cost consumers know there is a probability $1/(N - 1)$ that if they continue searching they may end up paying $2s$ before finding $p_L^*$ the ones that find it worthwhile to continue searching are those that have search costs smaller than $\frac{N-1}{N} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp$.

In the proof of the Proposition below we show that the first-order conditions with respect to $w_L$ and $w_H$ evaluated at the equilibrium prices yield

$$w_L^* D'(p_L^*(w_L)) \frac{\partial p_L}{\partial w_L} + D(p_L^*) \leq 0,$$

and

$$\left( 1 - \frac{1}{s} \int_{p_L(w_L)}^{p_H(w_H)} D(p) dp \right) \left[ w_H^* D'(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) \right] + \frac{1}{s} D(p_H^*) \frac{\partial p_H}{\partial w_H} \left[ w_L^* D(p_L^*) - w_H^* D(p_H^*) \right] = 0,$$

where $\frac{\partial p_L}{\partial w} =$
\[
\frac{[(N-1)\bar{s} + \hat{s}] D'(p^*_L) - \left( N - 1 + \frac{1}{N} \right) D^2(p^*_L)}{[(N-1)\bar{s} + \hat{s}] [2D'(p^*_L) + D''(p^*_L)(p^*_L - w^*_L)] - \left( N - 1 + \frac{1}{N} \right) D(\hat{p}_L) [3D'(\hat{p}_L)(\hat{p}_L - w_L) + 2D(\hat{p}_L)]}
\]

and
\[
\frac{\partial p_H}{\partial w_H} = \frac{D'(p^*_H)(\bar{s} - \hat{s}) - D^2(p^*_H)}{D(p^*_H) [3D'(p^*_H)(p_H - w_H) + D(p^*_H)]} = \frac{D'(p^*_H) (\bar{s} - \hat{s})}{D(p^*_H) [3D'(p^*_H)(p_H - w_H) + D(p^*_H)]}.
\]

Given that the equilibrium requires that
\[
w^*_H D(p^*_H) = w^*_L D(p^*_L),
\]
the first-order condition with respect to \(w_H\) can be simplified to
\[
w^*_H D'(p^*_H) \frac{\partial p_H}{\partial w_H} + D(p^*_H) = 0.
\]

The next proposition shows that there does not exist an equilibrium where all the necessary conditions for an equilibrium to exist are satisfied.

**Proposition 6** An equilibrium with wholesale price discrimination requires that the equations (7), (12), (8) and (13) and the inequality (9) are satisfied. This is not the case for linear demand and \(\bar{s}\) is small enough.

The Proposition basically shows that the only way to satisfy the equal profit condition (12) and not to have an incentive to set a different high wholesale price (13) is for the manufacturer to set a low wholesale price \(w^*_L\), for which it has an incentive to deviate. Alternatively, the only way to guarantee that (13) is satisfied is when the manufacturer profit per consumer is higher at the low wholesale price \(w^*_L\) than at the high wholesale price \(w^*_H\). However, given that retailers do not observe the wholesale prices set to their competitors, the manufacturer would then be able to profitably and secretly deviate and set \(w^*_L\) to all her retailers. Numerical analysis shows that the conclusion of the Proposition also holds true for more general demand functions and for \(\bar{s}\) not being small. In Figure 4.1 we plot the necessary equilibrium conditions for the case of linear demand.
5 Requiring Sales at Recommended Price

In the previous section, we have shown that wholesale price discrimination cannot be an equilibrium outcome if the manufacturer needs to make the same profits over different retailers, i.e., the equal profit condition given in (12) needs to be satisfied. In the Introduction we have argued that the Code of Federal Regulations effectively imposes restrictions on the deviations the manufacturer may contemplate. In particular, by requiring that at least some consumers buy at the recommended retail price, the manufacturer may announce the high retail price $p^*_H$ as a recommended retail price, and is then effectively committed to make sure that at least some consumers buy at this price. This would imply that by contemplating wholesale price discrimination and announcing $p^*_H$ as a recommended retail price, the manufacturer is not allowed to set all retailers the same wholesale price $w^*_L$. Accordingly, the equal profit condition given in (12) would not need to hold and an equilibrium with wholesale price discrimination would only need to satisfy the conditions given in (7), (8), (9) and (13). Requiring sales at the recommended retail price, the wholesale price discriminating equilibrium that maximizes total surplus has these four equilibrium conditions holding with equality.

The next Proposition argues that in any equilibrium where there is effective wholesale price discrimination in the sense that different consumers buy at different retail prices, i.e., $0 < \bar{s} < \bar{s}$, prices converge to the efficient equilibrium in the uniform pricing case if $\bar{s} \to 0$. One can show that $p^*_H(w^*_H) \to w^*_H$ and that this implies that $w^*_L \to w^*_H$, while $\frac{\partial p^*_H(w^*_L)}{\partial w^*_L}$ reduces to $\frac{1}{2}$. The next Proposition states the result.
Proposition 7 If \( \pi \rightarrow 0 \) all equilibria with effective wholesale price discrimination converges to \( p_L^* = w_L^* = p_H^* = w_H^* \), where \( w_L^* = w_H^* = w^* \) solves \( \frac{1}{2} w^* D'(w^*) + D(w^*) = 0 \).

5.1 Linear Demand

For linear demand \( D(p) = 1 - p \), we now have that in all the equilibria with wholesale price discrimination the high equilibrium retail price should satisfy:

\[
\left[ \pi - (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}) \right] [1 - 2p_H^* + w_H^*] - (1 - p_H^*)^2(p_H^* - w_H^*) = 0 \tag{14}
\]

The low equilibrium retail price should satisfy:

\[
\left[ 6\pi + 3(p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}) \right] [1 - 2p_L^* + w_L^*] - 7(1 - p_L^*)^2(p_L^* - w_L^*) \leq 0 \tag{15}
\]

The manufacturer’s equilibrium low wholesale price should satisfy:

\[
-w_L^* \frac{\partial p_L}{\partial w_L} + 1 - p_L^* \leq 0 \tag{16}
\]

where:

\[
\frac{\partial p_L}{\partial w_L} = \frac{2\pi + (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2})}{2 [2\pi + (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}) + \frac{7}{3}(1 - p_L^*)^2]} \]

and the manufacturer’s high wholesale price should satisfy:

\[
\left( \pi - (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}) \right) \left[ 1 - p_H^* - w_H^* \frac{\partial p_H}{\partial w_H} \right] + (1 - p_H^*) \frac{\partial p_H}{\partial w_H} [w_L^* (1 - p_L^*) - w_H^* (1 - p_H^*)] = 0 \tag{17}
\]

where

\[
\frac{\partial p_H}{\partial w_H} = \frac{1}{2} + \frac{\frac{3}{2}(1 - p_H^*) (p_H^* - w_H^*)}{2 \left( \pi + \frac{3}{2}(1 - p_H^*)^2 - \frac{1}{2}(1 - p_L^*)^2 \right) - 3(1 - p_H^*) (p_H^* - w_H^*)}
\]

Our next Proposition shows how for the case of linear demand retail and wholesale prices are affected by a change in \( \pi \).

Proposition 8 When \( \pi \) is small enough, under wholesale price discrimination with linear demand, retail and wholesale prices are decreasing and expected consumer surplus is increasing in \( \pi \). In a neighbourhood of \( \pi = 0 \), expected consumer surplus converges to \( \frac{\pi}{18} \), \( \frac{dw_L}{d\pi} \approx -2 \), \( \frac{dp_L}{d\pi} \approx -5 \), \( \frac{dp_H}{d\pi} \approx -\frac{1}{3} \), \( \frac{dw_H}{d\pi} \approx -\frac{5}{3} \) and \( \frac{dECS}{d\pi} \approx \frac{8}{27} \).

Proposition 8, reveals that under wholesale price discrimination both wholesale and retail prices are decreasing in \( \pi \). However, while both retail prices are decreasing in \( \pi \), the
low retail and wholesale price decrease at a much faster pace compared to the high retail and wholesale prices. Furthermore, in comparison to Proposition (5), it can be seen that while the low retail price decreases at the same pace as the retail price under uniform pricing, that is not the case for the high retail price. Given that we have shown before that for as \( \bar{s} \to 0 \), prices under uniform and wholesale price discrimination converge to the same price, Proposition (8) reveals that in a neighbourhood of \( \bar{s} = 0 \), while the low retail price may be close to the uniform price, the high retail price will indeed be higher.

![Fig 5.1 Wholesale and Retail prices for different values of \( \bar{s} \)](image)

For larger values of \( \bar{s} \) we can solve numerically for the equilibrium under wholesale price discrimination. Figure 5.1 shows how wholesale and retail prices change for different values of \( \bar{s} \). It can be seen that, under wholesale price discrimination, wholesale and retail prices are decreasing in \( \bar{s} \). The figure also confirms that when \( \bar{s} \to 0 \), retail margins are very small and that \( w^*_L \to \bar{w}^*_L \to w^* \to 2/3 \). The comparison of retail prices under wholesale price discrimination and uniform pricing is depicted in Figure 5.2. It is clear that under wholesale price discrimination, both the low and the high retail prices are larger than the retail price under uniform pricing. The comparison between wholesale prices is depicted in Figure 5.3. This figure reinforces Figure 5.2 that also the high wholesale prices that the manufacturer charges in the equilibrium under wholesale price discrimination are higher compared to the wholesale price of the uniform pricing case.
As was mentioned in the introduction, wholesale price discrimination acts as a mechanism that indirectly screens searching consumers. Consumers differing in their search costs react differently to retail prices. A low search cost consumer that observes a high retail price continue to search, while others stop and buy.

As a consequence, retailers do not face the same composition of search costs among their consumers. Specifically, low cost retailers’ demand consists of a relatively larger share
of low search cost consumers. Since low search cost consumers are more price sensitive, they will induce more competition between low cost retailers. In addition, because of the increased competition between the low cost retailers, consumers with higher search cost may also find it attractive to continue searching for lower prices forcing the high cost retailer also to lower its margins. Thus, both low and high cost retailers have lower margins under wholesale price discrimination as shown in Figure 5.4 below.

![Fig 5.4 Retail margins for different values of \( \sigma \)](image1)

On the other hand, Figure 5.5 shows the difference in profits between the uniform
pricing case and the price discrimination setting. These numerical results show that despite the lower margins, the low cost retailer earns higher profits compared to a retailer under uniform pricing for smaller values of \( \sigma \). The reason is that the difference in margins is small, while low cost retailers gain more sales due to low cost searchers that first visited the high cost retailers continuing to search for the low cost retailers. For larger values of \( \sigma \), the numerical analysis shows that it is the lower margins that dominate the impact on the low cost retailers’ profits. The profit of retailers under uniform pricing are always higher than the profit the high cost retailer makes under wholesale price discrimination.

Given the negative impact on consumer welfare, it is important to understand if the manufacturer has an incentive to engage in wholesale price discrimination. This will be the case if the manufacturer earns higher profits compared to uniform pricing. We can perform a similar analysis as with the other variables of interest. At \( \sigma = 0 \) the manufacturer makes the same profit whether or not it engages in wholesale price discrimination. When \( \sigma \) increases, the change in manufacturer profit under uniform pricing is given by:

\[
\frac{d\Pi_M}{d\sigma} = \frac{dw^*}{ds}(1 - p^*) - w^* \frac{dp^*}{ds},
\]

Fig 5.6 Manufacturer’s Profit for different values of \( \sigma \)

whereas we have a similar expression for the profit change per retailer under wholesale price discrimination. Using the above Propositions we can evaluate that as \( \sigma \to 0 \), the first-order approximation for the change in manufacturer profits will be \(-\frac{1}{3}\) for both uniform pricing and wholesale price discrimination. The second-order approximations of manufacturer profits reveal that the manufacturer is indeed better off when he engages in
wholesale price discrimination. For larger values of $\bar{\sigma}$ we can solve numerically. Manufacturer’s profit under both pricing practices are depicted in Figure 5.6. From the Figure it is clear that the manufacturer earns higher profit under wholesale price discrimination, but that the difference is small for smaller values of $\bar{\sigma}$.

Finally, since wholesale price discrimination leads to increased retail prices downstream this implies that consumer surplus will suffer. Under such a pricing practice, both low and high search cost consumers end up paying higher retail prices. Furthermore, a fraction of consumers with low search costs has to incur a search cost to find the low retail price $p^*_L$, while under uniform pricing consumer pay lower retail prices and do not have to incur a search cost. Results regarding expected consumer surplus from propositions (5) and (8), state that while expected consumer surplus is increasing in $\bar{\sigma}$, both under uniform and wholesale price discrimination, it increases twice as fast when the manufacturer sets uniform prices to his retailers. Figure 5.7 below, shows the difference in consumer surplus under these two different practices for larger values of $\bar{\sigma}$. From the figure we can see that the impact of wholesale price discrimination on consumer surplus can be quite large. For instance, for a search cost of 0.04, consumer surplus under wholesale price discrimination decreases by approximately 6%.

Fig 5.7 Expected Consumer Surplus for different values of $\bar{\sigma}$

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8Calculations available upon request.
6 Conclusion

In this paper, we focus on a vertically related industry with heterogeneous consumer search and analyse the impact of legislation which requires that a substantial number of sales are made at recommended retail prices. Despite the fact that competition authorities impose such restrictions with the aim of protecting consumers, we have shown that they may actually lead to the opposite effect. We have argued that such a constraint enables a monopolist manufacturer to effectively engage in wholesale price discrimination, which otherwise would not be possible. This occurs since the legal requirement serves as a commitment device and eliminates the possibility of the manufacturer to deviate and charge all retailers the low wholesale price.

We have shown that once the monopolist manufacturer has the possibility to price discriminate among his retailers he will charge low prices to some retailers and high prices to others. As retailers optimally react to such wholesale prices, the downstream market will consist of low and high retail prices. Given that consumers differ in their search costs, some of them will stop and buy at the their first search, while consumer that have lower search costs can afford to continue searching and only buy at a low retail price. Therefore, the demand of high cost retailers will consist of only high search cost consumers, while the low cost retailers’ demand will be made of a relatively larger share of low search cost consumers. In contrast, under uniform pricing retailers would each face the same demand composition. Thus, wholesale price discrimination acts as a mechanism that indirectly screens consumers according to their search costs.

The low search cost consumers, which are more price sensitive, increase competition between the low cost retailers, which makes even high search cost consumers more inclined to search. As a result, under wholesale price discrimination both types of retailers have lower margins. We have shown that as the upper bound of the search cost distribution increases, this lower margin effect dominates the impact on retailers’ profits and leads to lower profits for both, the high and the low cost, retailers. Furthermore, we have shown that the manufacturer is better off if he engages in wholesale price discrimination, increasing the average wholesale and retail prices. Finally, given the increased retail prices, consumer are worse off since, no matter their search cost, consumers end up buying at higher retail prices and some of them have to search twice.
7 Appendix

Proposition 3. The uniform pricing equilibrium that maximizes total surplus is given by \((1)\) and \((2)\), both holding with equality, where \(\frac{\partial \tilde{p}(w^*)}{\partial w}\) is given by \((4)\).

**Proof:** Under uniform wholesale pricing, \(p^*(w)\) is determined by \((1)\). It is clear that for any \(w < 1\) this expression is positive at \(p^* = w\) and negative at \(p^* = 1\). Moreover, as the derivative of the LHS wrt \(p^*\) can be written as \(-\frac{1}{w}[D(p^*)^2 - 2D(p^*)(p^* - w)] + 2D'(p^*) + D''(p^*)(p^* - w)\) and the part in square brackets is positive at \(p^* = w\) and 0 at \(p^* = 1\), while the remaining part is negative for most demand functions, it follows there must be a unique solution for \(p^*\) in the relevant interval \((w, 1)\).

To determine the equilibrium level of \(w\) we first have to consider how a retailer reacts to a deviation in \(w\). Assuming passive beliefs, an individual retailer will react by setting \(\tilde{p}\) such that \((3)\) is satisfied. Next, we show how we derived \(\frac{\partial \tilde{p}(w^*)}{\partial w}\). From \((3)\) it follows that:

\[
-2D(\tilde{p})(\tilde{p} - w)\frac{\partial \tilde{p}}{\partial w} - D^2(\tilde{p})\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) - D(\tilde{p})\left(D'(\tilde{p})(\tilde{p} - w) + D(\tilde{p})\right)\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) + \\
\left(\pi - \left(\int_{\tilde{p}}^{1} D(p)dp - \int_{\tilde{p}}^{1} D(p)dp\right)\right)\left(D''(\tilde{p})(\tilde{p} - w) + D'(\tilde{p})\left(\frac{\partial \tilde{p}}{\partial w} + D'(\tilde{p})\right)\right)\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) = 0,
\]

or

\[
-2D(\tilde{p})D'(\tilde{p})(\tilde{p} - w)\frac{\partial \tilde{p}}{\partial w} - D^2(\tilde{p})\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) - D(\tilde{p})\left(D'(\tilde{p})(\tilde{p} - w) + D(\tilde{p})\right)\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) + \\
\frac{D^2(\tilde{p})(\tilde{p} - w)}{D'(\tilde{p})(\tilde{p} - w) + D(\tilde{p})}\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) + D'(\tilde{p})\left(\frac{\partial \tilde{p}}{\partial w} + D'(\tilde{p})\right)\left(\frac{\partial \tilde{p}}{\partial w} - 1\right) = 0.
\]

Thus, using the fact that we want to evaluate \(\frac{\partial \tilde{p}(w^*)}{\partial w}\) at \(w = w^*\) and that in that case \(\tilde{p}(w^*) = p^*(w^*)\) we can use \((1)\) to get the expression in \((4)\).

**Proposition 4.** When \(\pi \to 0\) the uniform pricing equilibrium converges to \(p^* = w^*\), where \(w^*\) solves \(\frac{1}{2}w^*D'(w^*) + D(w^*) = 0\).

**Proof:** If \(\pi \to 0\) the expression for \(\frac{\partial \tilde{p}(w^*)}{\partial w}\) reduces to \(\frac{1}{2}\).

**Proposition 5.** When \(\pi\) is small enough, under uniform pricing with linear demand both \(p^*\) and \(w^*\) are decreasing in \(\pi\) with \(p^*\), while expected consumer surplus is increasing in \(\pi\). In a neighbourhood of \(\pi = 0\), we have that expected consumer surplus converges to \(\frac{1}{18}, \frac{dw^*}{d\pi} = -2, \frac{dE\text{SC}}{d\pi} = -5\) and \(\frac{dE\text{SC}}{d\pi} = \frac{2}{3}\).
Proof: (5) can be rewritten as
\[
\frac{\pi}{1 - p^*} = \frac{(1 - p^*)(p^* - w^*)}{(1 - p^*) - (p^* - w^*)}.
\]
Substituting into (6) gives
\[
(1 - p^*) \left[ 1 - w^* \frac{1 + \frac{(p^* - w^*)}{(1 - p^*) - (p^* - w^*)}}{2(1 - p^*) - 3(p^* - w^*) + \frac{2(1 - p^*)(p^* - w^*)}{(1 - p^*) - (p^* - w^*)}} \right] = 0,
\]
which can be rewritten as
\[
(1 - p^*) \left[ 1 - w^* \frac{1 - p^*}{2(1 - p^*) - 3(p^* - w^*)[(1 - p^*) - (p^* - w^*)] + 2(1 - p^*)(p^* - w^*)} \right] = 0.
\]
Thus, (6) holds true if, and only if,
\[
2(1 - p^*)^2 - 3(1 - p^*)(p^* - w^*) + 3(p^* - w^*)^2 - (1 - p^*)w^* = 0.
\]
Taking the total differential of this equation yields
\[
[-7(1 - p^*) + 9(p^* - w^*) + w^*] dp^* + [2(1 - p^*) - 6(p^* - w^*)] dw^* = 0,
\]
or
\[
\frac{dp^*}{dw^*} = \frac{-2(1 - p^*) + 6(p^* - w^*)}{-7(1 - p^*) + 9(p^* - w^*) + w^*},
\]
which in a neighbourhood of \( \pi = 0 \) (where \( p^* \approx w^* \approx \frac{2}{3} \)) is approximately equal to \( \frac{2}{5} \).

Similarly, we can rewrite (5) as
\[
\pi [1 - 2p^* + w^*] - (1 - p^*)^2(p^* - w^*) = 0
\]
and take the total differential to get
\[
[1 - 2p^* + w^*] d\pi + \left[ \pi + (1 - p^*)^2 \right] dw^* - \left[ 2\pi + (1 - p^*)^2 - 2(1 - p^*)(p^* - w^*) \right] dp^* = 0.
\]
In a neighbourhood of \( \pi = 0 \) (where \( p^* \approx w^* \approx \frac{2}{3} \)) this is approximately equal to
\[
\frac{1}{3} d\pi + \frac{1}{9} dw^* - \frac{1}{9} dp^* \approx 0.
\]
Using \( \frac{dp^*}{dw^*} \) is approximately equal to \( \frac{2}{5} \), we get that \( \frac{dw^*}{d\pi} \approx -5 \) and \( \frac{dp^*}{d\pi} \approx -2 \).

On the other hand, under uniform pricing with linear demand expected consumer surplus equals:
\[
ECS = \int_{p^*}^{1} D(p) dp = \frac{(1 - p^*)^2}{2}
\]

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From Proposition (4) we know that if $\bar{s} \to 0$ then $p^* \to \frac{2}{3}$. This and the above equation give that $ECS \to \frac{1}{18}$. In addition, taking the total differential of the $ESC$ under uniform pricing we obtain:

$$dECS = -(1 - p^*) dp^*$$

which we can write as:

$$\frac{dECS}{ds} = -(1 - p^*) \frac{dp^*}{ds}$$

Making use of Proposition (4), which shows that if $\bar{s} \to 0$, then $p^* \to \frac{2}{3}$ and the fact that if $\bar{s} \to 0$, then $\frac{dp^*}{ds} \to -2$, we obtain that in a neighbourhood of $s = 0$:

$$\frac{dECS}{ds} = -(1 - \frac{2}{3})(-2) = \frac{2}{3}$$

**Proposition 6.** For linear demand an unrestricted equilibrium with wholesale price discrimination does not exist when $s$ is small.

**Proof:** If an equilibrium would exist we should have that the following conditions holds. First, the FOC with respect to $p_L$:

$$- \left( \frac{N - 1 + \frac{1}{N}}{N - 1} \right) D^2(p_L^*) (p_L^* - w_L^*) \right] + D(p_L^*) (p_L^* - w_L^*) + D(p_L^*) = 0. \quad (18)$$

Second, the first-order condition with respect to $p_H$:

$$- \frac{D^2(p_H^*) (p_H^* - w_H^*)}{\bar{s} - \hat{s}} + \left[ D'(p_H^*) (p_H^* - w_H^*) + D(p_H^*) \right] = 0. \quad (19)$$

Third, the first-order condition with respect to $w_H$ evaluated at the equilibrium prices:

$$\left(1 - \frac{1}{\bar{s}} \int_{p_H^*}^{p_H^*} D(p) dp \right) \left[ w_H^* D'(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) \right] + \frac{1}{\bar{s}} \frac{\partial p_H}{\partial w_H} \left[ w_L^* D(p_L^*) - w_H^* D(p_H^*) \right] = 0,$n

where

$$\frac{\partial p_H}{\partial w_H} = \frac{D'(p_H^*) (\bar{s} - \hat{s}) - D^2(p_H^*)}{-D(p_H^*) \left[ 3D'(p_H^*) (p_H - w_H) + D(p_H^*) \right] + (\bar{s} - \hat{s}) \left( 2D'(p_H^*) + D'(p_H^*) (p_H - w_H) \right) - D^2(p_H^*)}, \quad (19)$$

Finally, the equilibrium requires that

$$w_H^* D(p_H^*) = w_L^* D(p_L^*), \quad (20)$$

We will show that these four conditions imply that the first-order condition with respect to $w_L$ is positive, i.e.,

$$w_L^* D'(p_L^* (w_L)) \frac{\partial p_L}{\partial w_L} + D(p_L^*) > 0, \quad (21)$$
Thus, we have that
\[
\frac{\partial p_H}{\partial w_H} = \frac{\left( (N - 1)\bar{s} + \bar{s} \right) D'(p^*_H) - \left( (N - 1 + \frac{1}{N}) D^2(p^*_L) \right)}{\left( (N - 1)\bar{s} + \bar{s} \right) \left[ 2D'(p^*_L) + D''(p^*_L)(p^*_L - w_L) \right] - \left( (N - 1 + \frac{1}{N}) D(p^*_L) \left[ 3D'(p^*_L)(p^*_L - w_L) + 2D(p^*_L) \right] \right]}.
\]

Using (12), it is clear that the first-order condition with respect to \( w_H \) can be simplified to
\[
w_H^* D'(p^*_H) \frac{\partial p_H}{\partial w_H} + D(p^*_H) = 0, \tag{23}
\]
or
\[
w_H^* \frac{\partial p_L}{\partial w_L} < - \frac{1}{D'(p^*_L)}.
\]

As (16) can be rewritten as
\[
w_L^* \frac{\partial p_L}{\partial w_L} < \frac{w_H^* \frac{\partial p_H}{\partial w_H}}{D(p^*_L) \frac{\partial p_H}{\partial w_H}},
\]

which under the indifference condition \( w_H^* D(p^*_H) = w_L^* D(p^*_L) \) can be rewritten as
\[
\frac{D(p^*_H) \frac{\partial p_L}{\partial w_L}}{D(p^*_L) \frac{\partial p_H}{\partial w_H}} < \frac{D(p^*_L) \frac{\partial p_H}{\partial w_H}}{D(p^*_H) \frac{\partial p_L}{\partial w_L}}.
\]

As \( D(p^*_H) < D(p^*_L) \) this is certainly true if \( \frac{\partial p_L}{\partial w_L} < \frac{\partial p_H}{\partial w_H} \). For linear demand (with slope \(-\beta\)),
\[
\frac{\partial p_L}{\partial w_L} = -\beta \left[ (N - 1)\bar{s} + \bar{s} \right] - \left( N - 1 + \frac{1}{N} \right) D^2(p^*_L)
\]
and
\[
\frac{\partial p_H}{\partial w_H} = -\beta (\bar{s} - \bar{s}) - D^2(p^*_H) \left[ -3\beta(p^*_H - w_H) + 2D(p^*_H) \right].
\]

Thus, we have that \( \frac{\partial p_L}{\partial w_L} < \frac{\partial p_H}{\partial w_H} \) if, and only if,
\[
3\beta^2 \left( N - 1 + \frac{1}{N} \right) (\bar{s} - \bar{s}) D(p^*_L)(p^*_L - w_L)
< 3\beta^2 \left[ (N - 1)\bar{s} + \bar{s} \right] D(p^*_H)(p^*_H - w_H) + 3 \left( N - 1 + \frac{1}{N} \right) \beta D(p^*_L)D(p^*_H) (D(p^*_L) - D(p^*_H))(p^*_L - w_L).
\]
As for linear demand $D(p^*_L) - D(p^*_H) = \beta (p^*_H - p^*_L)$, this inequality can be rewritten as

\[
\left( N - 1 + \frac{1}{N} \right) (\widehat{s} - \check{s}) D(p^*_L)(p^*_L - w_L) < \left( N - 1 + \frac{1}{N} \right) D(p^*_L)D(p^*_H) (p^*_H - p^*_L)(p^*_L - w_L).
\]

(24) can be rewritten as

\[
\left[ (N - 1)\check{s} + \widehat{s} \right] D(p^*_L)(p^*_L - w_L) + \left( N - 1 + \frac{1}{N} \right) D(p^*_L)D(p^*_H) (p^*_H - p^*_L)(p^*_L - w_L).
\]

As the FOC for $p_L$ and $p_H$ can be rewritten as

\[
(N - 1)\check{s} + \widehat{s} = \frac{(N - 1 + \frac{1}{N}) D^2(p^*_L)(p^*_L - w_L)}{-\beta(p^*_L - w_L) + D(p^*_H)}.
\]

and

\[
\check{s} - \widehat{s} = \frac{D^2(p^*_L)(p^*_L - w_L)}{-\beta(p^*_L - w_L) + D(p^*_H)}.
\]

(24) can be rewritten as

\[
\frac{D(p^*_L)(p^*_L - w_H)}{-\beta(p^*_L - w_H) + D(p^*_H)} < \frac{D(p^*_L)(p^*_L - w_H)}{-\beta(p^*_L - w_L) + D(p^*_L)} + (p^*_L - p^*_L),
\]

or

\[
-\beta(p^*_L - w_H) [D(p^*_L) - (p^*_L - w_H)D(p^*_L)] < (p^*_L - p^*_L) \left[ D(p^*_H) - \beta(p^*_L - w_H) \right] [D(p^*_L) - \beta(p^*_L - w_L)].
\]

As in a neighborhood of $\check{s} = 0$ $p^*_H \approx p^*_L \approx w^*_L \approx w^*_H$, the LHS is of an order smaller than the RHS.

**Proposition 7.** If $D''(p) \leq 0$, then if $\check{s} \to 0$ all equilibria with effective wholesale price discrimination converges to $p^*_L = w^*_L = p^*_H = w^*_H$, where $w^*_L = w^*_H = w^*$ solves

\[
\frac{1}{2} w^* D'(w^*) + D(w^*) = 0.
\]

**Proof.** Consider first (7) if $\check{s} \to 0$. As also $\widehat{s} \to 0$, and $D'(p^*_H) < 0$ while $D(p^*_H) > 0$ it must be the case that in any equilibrium with wholesale price discrimination $p^*_H \to w^*_H$. As $0 < \check{s} < \check{s}$, where $\check{s} = \frac{p^*_L}{p^*_L} D(p) dp$, it must be the case that $p^*_L \to p^*_H$ if $\check{s} \to 0$. Next, consider (8) if $\check{s} \to 0$. Since also $\widehat{s} \to 0$, and $D'(p^*_H) < 0$ while $D(p^*_L) > 0$ it must be that in any equilibrium with wholesale price discrimination $p^*_L \to w^*_L$. Thus, if $\check{s} \to 0$ then it follows that $p^*_H \approx p^*_L \approx w^*_H \approx w^*_L$. It remains to be seen to which values the wholesale and retail prices converge. To this end, consider (10) in a neighbourhood of $\check{s} = 0$ where $p^*_L - w^*_L = 0$. It is easy to see that

\[
\frac{\partial p_L}{\partial w_L} \approx \frac{-(N - 1 + \frac{1}{N}) D^2(p^*_L)}{-2(N - 1 + \frac{1}{N}) D^2(p^*_L)} \approx \frac{1}{2}.
\]

Thus, in a neighbourhood of $\check{s} = 0$ the first-order condition determining $w^*_L$ can be simplified to

\[
\frac{1}{2} w^*_L D'(w^*_L) + D(w^*_L) \approx 0.
\]
**Proposition 8.** When \( \bar{s} \) is small enough, under wholesale price discrimination with linear demand, retail and wholesale prices are decreasing in \( \bar{s} \) and expected consumer surplus is increasing in \( \bar{s} \). In a neighbourhood of \( \bar{s} = 0 \), expected consumer surplus converges to \( \frac{1}{18}, \frac{d\bar{s}}{d\bar{s}} \approx \frac{d\bar{s}}{d\bar{s}} \approx -2, \frac{dp^*_H}{d\bar{s}} \approx -5, \frac{dp^*_L}{d\bar{s}} \approx -\frac{1}{3}, \frac{d\bar{s}}{d\bar{s}} \approx -\frac{5}{3} \) and \( \frac{dECS}{d\bar{s}} \approx \frac{8}{27} \).

**Proof:** Equation (15) can be rewritten as:

\[
2\bar{s} + (p^*_H - p^*_L)(1 - \frac{p^*_H + p^*_L}{2}) = \frac{7}{3}(1 - p^*_L)^2(p^*_L - w^*_L)
\]

Substituting into (16) gives:

\[
1 - p^*_L - w^*_L \left[ \frac{7}{3}(1 - p^*_L)(p^*_L - w^*_L) + \frac{7}{3}(1 - 2p^*_L + w^*_L) \right] = 0
\]

or

\[
1 - w^*_L \left[ \frac{7}{3}(1 - p^*_L)(p^*_L - w^*_L) + (2 + 3w^*_L - 5p^*_L)(1 - 2p^*_L + w^*_L) \right] = 0
\]

Thus, (16) holds true if, and only, if:

\[
2(1 - p^*_L)(p^*_L - w^*_L) + (2 - 2p^*_L + w^*_L)(2 + 3w^*_L - 5p^*_L) - w^*_L(1 - p^*_L) = 0
\]

Taking the total differential of this equation yields:

\[
0 = [(2 + 3w^*_L - 5p^*_L) + 3(1 - 2p^*_L + w^*_L) - 3(1 - p^*_L)] dw^*_L
- [2(p^*_L - w^*_L) - 2(1 - p^*_L) + 5(1 - 2p^*_L + w^*_L) + 2(2 + 3w^*_L - 5p^*_L) - w^*_L] dp^*_L
\]

which in a neighbourhood of \( \bar{s} = 0 \) (where \( p^*_L \approx w^*_L \approx \frac{2}{3} \)) reduces to \( \frac{2}{3}dw^*_L - \frac{5}{3}dp^*_L = 0 \).

Similarly, we can rewrite (15) as:

\[
2\bar{s} + (p^*_H - p^*_L)(1 - \frac{p^*_H + p^*_L}{2}) [1 - 2p^*_L + w^*_L] - \frac{7}{3}(1 - p^*_L)^2(p^*_L - w^*_L) = 0
\]

and take the total differential to get:

\[
0 = [2(1 - 2p^*_L + w^*_L)] ds + \left[ 2\bar{s} + (p^*_H - \frac{(p^*_H)^2}{2}) + \frac{(p^*_L)^2}{2} - p^*_L \right] dw^*_L + [1 - p^*_H] dp^*_H + \left[ -2(p^*_H - \frac{(p^*_H)^2}{2}) + \frac{(p^*_L)^2}{2} - p^*_L \right] d\bar{s}
\]

In a neighbourhood of \( \bar{s} = 0 \) (where \( p^*_L \approx w^*_L \approx \frac{2}{3} \)) this is approximately equal to:

\[
\frac{2}{3}d\bar{s} - \frac{10}{27}dp^*_L + \frac{7}{27}dw^*_L + \frac{1}{3}dp^*_H = 0
\]

On the other hand we have that the expression:

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\[ \frac{\partial p_H}{\partial w_H} = \frac{1}{2} + \frac{\frac{3}{2}(1 - p_H^*) (p_H^* - w_H^*)}{2 \left( \bar{\sigma} - \frac{3}{2}(1 - p_H^*)^2 - \frac{1}{2}(1 - p_L^*)^2 \right) - 3(1 - p_H^*) (p_H^* - w_H^*)} \]

can be rewritten as:

\[ \frac{\partial p_H}{\partial w_H} = \frac{(1 - p_H^*)^2 + (\bar{\sigma} - (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}))}{(1 - p_H^*)(2 - 5p_H^* + 3w_H^*) + 2(\bar{\sigma} - (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}))} \]

On the other hand, (14) can be rewritten as:

\[ \bar{\sigma} - (p_H^* - p_L^*)(1 - \frac{p_H^* + p_L^*}{2}) = \frac{(1 - p_H^*)^2(p_H^* - w_H^*)}{(1 - 2p_H^* + w_H^*)} \]

substituting it into (17) gives:

\[ \frac{(1 - p_H^*)^2(p_H^* - w_H^*)}{(1 - 2p_H^* + w_H^*)} \left( 1 - p_H^* - w_H^* \frac{\partial p_H}{\partial w_H} \right) + \frac{\partial p_H}{\partial w_H} \left[ w_L^*(1 - p_L^*) - w_H^*(1 - p_H^*) \right] = 0 \]

which can be rewritten as:

\[ \frac{(1 - p_H^*)(p_H^* - w_H^*)}{(1 - 2p_H^* + w_H^*)} \left( 1 - p_H^* - w_H^* \frac{\partial p_H}{\partial w_H} \right) + \frac{\partial p_H}{\partial w_H} \left[ w_L^*(1 - p_L^*) - w_H^*(1 - p_H^*) \right] = 0 \]

or:

\[ (p_H^* - w_H^*) - \frac{(w_H^*(1 - p_H^*)^2 - w_L^*(1 - p_L^*)(1 - 2p_H^* + w_H^*))}{(2 - 5p_H^* + 3w_H^*)(1 - 2p_H^* + w_H^*) + 2(1 - p_H^*)(p_H^* - w_H^*)} = 0 \]

Thus, (17) holds true if and only if:

\[ (p_H^* - w_H^*) \left[ (2 - 5p_H^* + 3w_H^*)(1 - 2p_H^* + w_H^*) + 2(1 - p_H^*)(p_H^* - w_H^*) \right] - \left[ w_H^*(1 - p_H^*)^2 - w_L^*(1 - p_L^*)(1 - 2p_H^* + w_H^*) \right] = 0 \]

The total differential of this equation in neighbourhood of \( \bar{\sigma} = 0 \) yields:

\[ -\frac{1}{9}dw_H^* + \frac{2}{9}dp_H^* - \frac{2}{9}dp_L^* + \frac{1}{9}dw_L^* = 0 \]

which can be rewritten as:

\[ dw_H^* - 2dp_H^* = dw_L^* - 2dp_L^*. \]
Similarly, we can rewrite (14) as:
\[
\left[ s - \left( p_H^* - p_L^* \right) \left( 1 - \frac{p_H^* + p_L^*}{2} \right) \right] \left[ 1 - 2p_H^* + w_H^* \right] - (1 - p_H^*)^2(p_H^* - w_H^*) = 0
\]
and taking the total differential we obtain:
\[
0 = \left[ (1 - 2p_H^* + w_H^*) \right] ds + \left[ s - \left( p_H^* - \frac{(p_H^*)^2}{2} + \frac{(p_L^*)^2}{2} - p_L^* \right) + (1 - p_H^*)^2 \right] dw_H^* + \\
\left[ 2(p_H^* - \frac{(p_H^*)^2}{2} + \frac{(p_L^*)^2}{2} - p_L^* - s) - (1 - 2p_H^* + w_H^*) - 2(1 - p_H^*)(p_H^* - w_H^*) \right] dp_H^* + \\
\left[ (1 - p_L^*) \left( 1 - 2p_H^* + w_H^* \right) \right] dp_L^*
\]
which in a neighbourhood of \( s = 0 \) (where \( p_H^* \approx w_H^* \approx \frac{2}{3} \)) approximately equals to:
\[
\frac{1}{3}ds + \frac{1}{9}dp_L^* - \frac{2}{9}dp_H^* + \frac{1}{9}dw_H^* = 0
\]
Therefore, we have a system of four equations with four unknowns, which can be rewritten as:
\[
2 \frac{dw_L^*}{ds} - 5 \frac{dp_L^*}{ds} = 0
\]
\[
7 \frac{dw_L^*}{ds} - 10 \frac{dp_L^*}{ds} + 9 \frac{dp_H^*}{ds} = -18
\]
\[
\frac{dw_L^*}{ds} - 2 \frac{dp_L^*}{ds} - \frac{dw_H^*}{ds} + 2 \frac{dp_H^*}{ds} = 0
\]
\[
\frac{dp_L^*}{ds} + \frac{dw_H^*}{ds} - 2 \frac{dp_H^*}{ds} = -3
\]
Solving this system we obtain: \( \frac{dw_L^*}{ds} = -5, \frac{dp_L^*}{ds} = -2, \frac{dw_H^*}{ds} = -\frac{5}{3} \) and \( \frac{dp_H^*}{ds} = -\frac{1}{3} \).

On the other hand, under wholesale price discrimination expected consumer surplus equals:
\[
ECS = \frac{1}{3} \left( 1 - \frac{1}{3} \int_0^{p_H^*} D(p)\,dp + \frac{2}{3} \int_{p_H^*}^{1} D(p)\,dp + \frac{1}{3} \int_0^{p_H^*} D(p)\,dp + \int_{p_H^*}^{1} D(p)\,dp - E(s) \right)
\]
\[
= \frac{1}{3} \int_{p_H^*}^{1} D(p)\,dp + \frac{2}{3} \int_{p_H^*}^{1} D(p)\,dp - \frac{1}{3} \int_{p_H^*}^{p_H^*} D(p)\,dp + \frac{1}{3} \left( \int_{p_H^*}^{p_H^*} D(p)\,dp \right)^2
\]
\[
= \frac{1}{3} \int_{p_H^*}^{1} D(p)\,dp + \frac{2}{3} \int_{p_H^*}^{1} D(p)\,dp - \frac{1}{3} \int_{p_H^*}^{p_H^*} D(p)\,dp + \frac{1}{3} \left( \int_{p_H^*}^{p_H^*} D(p)\,dp \right)^2
\]
\[
= \frac{1}{3} \int_{p_H^*}^{1} D(p)\,dp + \frac{2}{3} \int_{p_H^*}^{1} D(p)\,dp - \frac{1}{3} \int_{p_H^*}^{p_H^*} D(p)\,dp + \frac{1}{3} \left( \int_{p_H^*}^{p_H^*} D(p)\,dp \right)^2
\]
\[
= \frac{1}{3} \int_{p_H^*}^{1} D(p)\,dp + \frac{2}{3} \int_{p_H^*}^{1} D(p)\,dp - \frac{1}{3} \int_{p_H^*}^{p_H^*} D(p)\,dp + \frac{1}{3} \left( \int_{p_H^*}^{p_H^*} D(p)\,dp \right)^2
\]
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For the case of linear demand the above expression becomes:

\[
\frac{1}{3}(1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + \frac{2}{3}(1-p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \frac{1}{3}(p_H^* - p_L^*) \left( 1 - \frac{p_H^* + p_L^*}{2} \right) + \frac{1}{3} s \left( p_H^* - p_L^* \right) \left( 1 - \frac{p_H^* + p_L^*}{2} \right)^2
\]

which can also be written as:

\[
\frac{\pi(1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + 2\pi(1-p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \pi(p_H^* - p_L^*) \left( 1 - \frac{p_H^* + p_L^*}{2} \right) + \left( p_H^* - p_L^* \right) \left( 1 - \frac{p_H^* + p_L^*}{2} \right)^2}{3}.
\]

From Proposition (7) we know that if \( \pi \to 0 \) then \( p_L^* \to p_H^* \to \frac{2}{3} \). Given that the above written expression, as \( \pi \to 0 \), yields the indeterminate form \( \frac{0}{0} \), we make use of L’Hospital’s Rule and obtain the following:

\[
(1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + 2(1-p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - (p_H^* - p_L^*) \left( 1 - \frac{p_H^* + p_L^*}{2} \right)
\]

Using Proposition (7) and the above equation we obtain that under wholesale price discrimination with linear demand if \( \pi \to 0 \) then \( ECS \to \frac{1}{18} \).

In addition, the ESC given in (26) can be rewritten as:

\[
0 = \pi(1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + 2\pi(1-p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \pi(p_H^* - p_L^*) \left( 1 - \frac{p_H^* + p_L^*}{2} \right) + \left[ (p_H^* - p_L^*) \left( 1 - \frac{p_H^* + p_L^*}{2} \right) \right]^2 - 3\pi ECS
\]

Taking the total differential of the above expression we obtain:

\[
\left[ -\pi \left( \frac{1}{2} - \frac{p_H^*}{2} \right) - \frac{1}{2}(\pi - \pi p_H^*) - \pi(1-p_H^*) + 2 \left( p_H^* - \frac{p_H^2}{2} - p_L^* + \frac{p_L^2}{2} \right) (1-p_H^*) \right] dp_H^*
\]

\[
+ \left[ -2\pi \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \frac{1}{2}(2\pi - 2\pi p_H^*) - \pi(p_L^* - 1) + 2 \left( p_H^* - \frac{p_H^2}{2} - p_L^* + \frac{p_L^2}{2} \right) (p_L^* - 1) \right] dp_L^*
\]

\[
+ \left[ (1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + (2 - 2p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \left( p_H^* - \frac{p_H^2}{2} - p_L^* + \frac{p_L^2}{2} \right) - 3ECS \right] ds
\]

\[-3\pi dECS = 0
\]

which we can write as:

\[
\frac{dECS}{ds} = \frac{(1-p_H^*) \left( \frac{1}{2} - \frac{p_H^*}{2} \right) + (2 - 2p_L^*) \left( \frac{1}{2} - \frac{p_L^*}{2} \right) - \left( p_H^* - \frac{p_H^2}{2} - p_L^* + \frac{p_L^2}{2} \right) - 3ECS}{3s}
\]

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\[
\left[ -2\pi \left( \frac{1}{2} - \frac{p^*_L}{2} \right) - \frac{1}{2} (2\pi - 2\pi p^*_L) - \pi (p^*_L - 1) + 2 \left( p^*_L - p^2_L - p^*_L + \frac{p^*_L}{2} \right) (p^*_L - 1) \right] \frac{dp^*_L}{d\pi}
\]

\[
\left[ -\pi \left( \frac{1}{2} - \frac{p^*_H}{2} \right) - \frac{1}{2} (\pi - \pi p^*_H) - \pi (1 - p^*_H) + 2 \left( p^*_H - p^2_H - p^*_L + \frac{p^*_L}{2} \right) (1 - p^*_H) \right] \frac{dp^*_H}{d\pi}
\]

From Proposition (7) we know that if \( \pi \to 0 \) then \( p^*_L \to p^*_H \to \frac{2}{3} \) and from what was proven above we know that if \( \pi \to 0 \), then \( \frac{dp^*_L}{d\pi} = -2 \) and \( \frac{dp^*_H}{d\pi} = -\frac{1}{3} \). Therefore, the above expression, as \( \pi \to 0 \), yields the indeterminate form \( \frac{0}{0} \). To overcome this we make use of L’Hospital’s Rule and obtain the following:

\[
\left[ -2 \left( \frac{1}{2} - \frac{p^*_L}{2} \right) - \frac{1}{2} (2 - 2p^*_L) - (p^*_L - 1) \right] \frac{dp^*_L}{d\pi} + \left[ - \left( \frac{1}{2} - \frac{p^*_H}{2} \right) - \frac{1}{2} (1 - p^*_H) - (1 - p^*_H) \right] \frac{dp^*_H}{d\pi}
\]

Therefore, we have that in a neighbourhood of \( \pi = 0 \):

\[
\frac{dECS}{d\pi} = \left( -\frac{1}{3} \right) (-2) + \left( -\frac{2}{3} \right) \left( -\frac{1}{3} \right) = \frac{8}{27}
\]
References


