Information, Authority, and Smooth Communication in Organizations

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July 2016

Abstract

We study a general sender-receiver game with endogenous information. A designer chooses the information that the sender gets to observe with a view to maximizing aggregate surplus. In a world with correlated interests and noisy signals, information creates and resolves conflicts. The optimum from the designer's perspective orthogonalizes the sender's preferred choice and the underlying conflicts between sender

*This paper is a spin-off of an earlier version, entitled "Smooth, strategic communication". We thank first and foremost Renato Gomes for his excellent discussion of our work. Many thanks also to Ricardo Alonso, Charles Angelucci, Andreas Blume, Patrick Bolton, Wouter Dessein, Peter Eső, Emir Kamenica, Botond Kőszegi, Matthias Lang, David Martimort, Meg Meyer, Benny Moldovanu, Stephen Morris, Inés Moreno de Barreda, Marco Ottaviani, Sven Rady, Joel Sobel, Peter Sørensen, Peter Vida, and seminar participants at the Universities of Aarhus, Bonn, CEU Budapest, Columbia, Copenhagen, Crest, Oxford, PSE, Zurich; SYME 2013 (Aarhus), ESEM 2014 (Toulouse), ESSET 2015 (Gerzensee), VfS 2014 (Hamburg), the SFB-TR15 conference 2014 (Mannheim), and the 2016 Toulouse conference in honor of Jacques Crémer. Financial support by the DFG in form of a SFB-TR15 grant and the BGSE is gratefully acknowledged. All remaining errors are our own.

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and receiver, enabling honest communication of the sender's unstrategically preferred choice. Communication and delegation of decision-rights become outcome equivalent.

JEL: D82
Keywords: information design, strategic communication, authority

1 Introduction

Sound decision making requires good information. The success of organizations depends crucially on the quality of information their decision-makers have. Most of the time, organizations do not have automatic access to information but must actively acquire it prior to decision-making. The search for information is subject to choices and must be considered as part of the decision-making process. How and where the information enters the organization is by and large determined by the organization's existing structure. Inside the organization the information needs to be communicated to the decision-maker. Such communication is prone to strategic manipulation; on the way towards the decision-maker, inferences are drawn, details can be dropped, things can be swept under the rug. The present paper tries to shed light on how organizations with given communication channels can cope with such problems. We show that, under certain conditions, an appropriate acquisition of information can ensure sound decision-making despite strategic communication.

We study a general sender-receiver game that involves three parties in fixed roles. A sender, who gets access to information, a receiver, who has the right to make a choice affecting the payoffs of the sender and the receiver, and a designer, who wishes to maximize aggregate surplus. The sole influence of the designer is the choice of what information the sender gets to observe. The information is nonverifiable; once the sender has obtained the information, he sends a message to the receiver. The receiver is unable to commit to an action policy ex ante. Thus, the situation corresponds to one of cheap talk in the spirit of Crawford and Sobel (1982) with information that is chosen endogenously by a designer. We are primarily interested in the role of information in shaping and resolving conflicts in the information transmission game. To isolate these effects, we assume that there are no exogenously given conflicts between the sender and the receiver: based on prior information alone, the sender and the receiver agree on the ideal action to take. Hence, all conflicts that arise between them ex post are due to the information that the sender was allowed to observe. While we take this as given for the main part of our analysis, we show later on that a simple transfer price scheme achieves precisely this outcome, where all foreseeable conflicts are eliminated and the only ones that remain arise ex post, after information has been obtained. We assume that the first-best optimal policies of the sender and the receiver correspond to the realizations of a pair of positively correlated random variables and that the sender gets to observe the realization of each random variable plus noise. The joint distribution of all random variables belongs to the elliptical class, so all conditional means are linear. The designer's instruments are the variances of the noise terms in the sender's observations.

We solve the designer's problem of optimal information provision as follows. We begin by showing that any equilibrium of the communication game is essentially equivalent to one with communication about the sender's conditionally expected mean. The reason is that this variable is a sufficient statistic for the sender's information from the sender's perspective. We then move on to derive the designer's maximization problem from first principles. To solve this problem, we first look at the clearly unrealistic case where the sender's conditionally expected state is publicly observable. This gives rise to a double regression problem for the receiver: the sender's conditional mean is a linear regression of his unknown state on the observed pair of signals and the receiver's conditional mean is a linear regression of the receiver's unknown state on the sender's conditional mean. The designer anticipates that the receiver chooses her optimal policy according to this double regression rule and anticipates that the sender and the receiver face residual risks after the receiver has chosen her optimal policy. The designer's problem then amounts to a risk sharing problem with an endogenous amount of total risk. Total risk is minimized by making the sender's conditional mean contain all common information. In contrast, idiosyncratic information is acquired to the point where residual risks to the sender and the receiver are equalized. The optimal information structure has the property that conditional on the sender's ideal policy, i.e. his conditional mean, the sender's and the receiver's underlying states are uncorrelated. By implication, the sender's ideal policy is orthogonal to the sender's bias in decision-making, i.e. the difference between the ideal policies of sender and receiver. This means that observability of the sender's conditional mean is inessential: the communication game with nonverifiable information has an equilibrium in which, for the optimal information structure, the sender truthfully announces his *unstrategically* preferred policy and the receiver follows his advice one-for-one. Since the sender's message strategy is differentiable in (the sufficient statistic of) his information and the receiver's action strategy is differentiable in the sender's message, we term this communication equilibrium a "smooth communication equilibrium".

The main insights from our analysis are as follows. Smooth communication is outcome equivalent to delegation. Since the informed party gets its way anyhow, it does not matter who has the formal authority to take decisions. Moreover, for the optimal information structure, smooth communication is even outcome equivalent to optimal delegation. If the sender were given the right to choose decisions, these decisions would be optimal against his information and the constraints imposed on his choices. However, if the sender uses his information in the receiver's best interest, then there is clearly no point in constraining his discretion. All in all, the design of the information structure serves as a substitute for adjusting the organization's structure in other ways such as the allocation of decision-rights.

In many ways, we reverse the perspective taken by classical mechanism design, which searches for optimal mechanisms in face of given information structures. Instead, we take the decision-mechanism as given and adjust the information structure in the game. Bergemann and Morris (2016) have recently coined the term information design for this reversed perspective. We view our result as a hypothetical ideal: under ideal circumstances, several decision-mechanisms are outcome equivalent. The result has a similar flavor as the full rent extraction result of Crémer and McLean (1985, 1988). Even if we do not expect that the result always holds precisely in practice - because some of the underlying assumptions are violated - it still serves as a useful benchmark.

Our outcome equivalence result clearly does not hold if the sender has an ex ante known bias as in Crawford and Sobel (1982). Interestingly, this does not mean that there are no systematic differences of opinion to begin with, but rather that they can be eliminated. We show in an extension how this can be achieved by a simple transfer-price between the sender and the receiver. Other assumptions we make may create the erroneous first impression of a knife-edge character of the result: our baseline model assumes symmetric prior uncertainty of sender and receiver and assumes that the designer maximizes an unweighted sum of their utilities. These symmetry assumptions simplify the analysis dramatically but can be dropped at a cost. Building on techniques we develop in companion work (Deimen and Szalay (2016)), we show that the decision-mechanisms remain equivalent if the priors are not too asymmetric and if the designer puts relatively more weight on the sender's utility.

Our main analysis employs techniques familiar from the analysis of strategic market interactions in the context of a communication problem. Information in market contexts is frequently analyzed in a Gaussian world with linear best replies (see Bergemann and Morris (2016) for a recent contribution and see Vives (1999), Vives (2008), and Pavan and Vives (2015) for surveys of the literature).[1] A smaller literature assumes the more general case of elliptical distributions, which contains the Normal distribution as a special case (see e.g. Rochet and Vila (1994) and Nöldeke and Tröger (2006)).[2] We are not aware of any work allowing at the same time for general payoff functions and elliptical distributions, a contribution of our work. However, the main difference to this literature is that agents in the market context observe private signals directly and then interact in the market. In our framework, a designer chooses the private signals that a sender gets to observe; after that the sender decides what information to pass on to the receiver; finally, the receiver makes a choice affecting all parties' payoffs. Thus, information is transmitted by strategic actors and there is one common action that affects all parties' payoffs.

The literature on "delegated expertise" analyzes information acquisition by a strategic expert. Demski and Sappington (1987) define an expert as a person who can acquire information while others cannot. In contrast to the present paper, among many other differences, communication is prohibitively costly in their work. Following this tradition, Crémer and Khalil (1992), Crémer et al. (1998), and Szalay (2009) study information acquisition in a procurement context allowing for monetary payments. Szalay (2005) studies a communication model with commitment to decision-rules with and without allowing for money payments. The essential difference to this literature is the absence of a commitment to decision-rules in the present paper, a natural assumption that the literature following Crawford and Sobel

[1] See Gordon and Nöldeke (2013) for an application of the linear Gaussian model to communication. Their approach assumes exogenous information coupled with noisy communication in the spirit of Blume et al. (2007) and lying costs in the spirit of Kartik (2009).

[2] See also Mailath and Nöldeke (2008) for an analysis of market breakdown in the adverse selection model using elliptical distributions.

(1982) maintains.[3] Only few papers have analyzed information acquisition in this context. Indeed, Sobel (2013) identifies information acquisition as one of the key open questions in this literature. One way to organize the recent contributions is by way of who decides on what information is acquired and how it reaches the decision-maker. Argenziano et al. (forthcoming) analyze the sender's incentives to acquire information under various forms of decision-making and show that communication can outperform delegation, because the expert acquires more precise information than the decision-maker would have acquired. In our companion paper (Deimen and Szalay (2016)) we study information acquisition by the sender in a Laplace-quadratic environment, a special case of the present one. While Argenziano et al. (forthcoming) show that decision-making mechanisms impact on the amount of information that is acquired, we emphasize the role of the decision-making mechanism in directing the expert's attention towards this or that source of information. Di Pei (2015) assumes the sender can acquire coarse information - that is, partition the state space at a cost - and shows that the implications for equilibrium communication are quite different from Crawford and Sobel (1982).[4][5] Ivanov (2010) analyzes receiver optimal partitional information structures in the Crawford and Sobel (1982) model. In this problem, the receiver chooses what information the sender should be allowed to see before communicating with the receiver.

With the exception of our companion work, our problem differs from these papers both with respect to the assumptions and the results. We take the perspective of a designer who maximizes the sum of the sender's and the receiver's expected utilities. Information and conflicts are two different things in the literature, which assumes the case of an ex ante known bias as in Crawford and Sobel (1982). In our paper, information and conflicts are intimately tied together; information creates and resolves conflicts. While the comparison of institutions features prominently in the literature, none of the previous papers obtains an

[3]See Lewis and Sappington (1997) and Gromb and Martimort (2007) for results on the organization of delegated expertise.

[4]See also Frug (2016) for an analysis of sequential information acquisition in a communication model.

[5]Kamenica and Gentzkow (2011) can be understood as information acquisition by a sender who is committed to pass on all the information that is acquired to the receiver. Hence, there is no strategic communication in their problem.

equivalence between the decision-making mechanisms.[6] This is in part due to the absence of ex ante known conflicts and in part due to the perspective we take. If we took the perspective of the receiver as in Ivanov (2010), then the solution would be very different from the one we obtain. In contrast, the same is not true if we took the sender's perspective, which is precisely what we do in our companion paper (Deimen and Szalay (2016)). Last but not least, the problems all differ with respect to the information acquisition technologies; to the best of our knowledge, we are the first to study the elliptical family in the context of information acquisition and communication.[7]

In focusing on contributions that emphasize the role of information we miss out on the influential theory of organizations where the comparison of institutions originates. We devote an entire section to this discussion. However, since some connections are much easier to grasp after having seen the model, we postpone this discussion till Section six.

The remainder of the paper is organized as follows. In Section two, we present the model. In Section three, we analyze communication and derive an upper bound on the amount of information that can be transmitted in any equilibrium. In Section four, we analyze optimal information acquisition from the organization's perspective. In Section five we drop some convenient assumptions and demonstrate the robustness of our results. Section six is devoted to the literature on organizations. A final section concludes. Lengthy proofs are gathered in the Appendix.

2 Model

We consider a strategic interaction between three parties in fixed roles, a sender, a receiver, and a designer. E.g., the sender and the receiver could correspond to divisions of a firm and the designer could correspond to the headquarters of the firm. A decision $x \in \mathbb{R}$ needs to be

[6] See also Goltsman et al. (2009) for a comparison of delegation, communication, and mediation.

[7] A related question concerns the consequences of more precise information in communication games. Moscarini (2007) shows that a better informed sender can achieve better communication. Szalay (2012) shows that better information results in a higher variability of choices. Ottaviani and Sørensen (2006) study communication by a sender who wishes to appear well informed.

taken that affects the payoffs of all three parties. The sender has preferences described by

$$u^S(x,\eta) = -\ell(x-\eta);$$

the receiver has preferences

$$u^R(x,\omega) = -\ell(x-\omega).$$

The loss function $\ell(q)$ is symmetric around its minimizer, $q=0$, twice differentiable, and at least as convex as the quadratic function. More precisely, we assume that the Arrow-Pratt measure of relative curvature of the loss function satisfies $\frac{q\ell''(q)}{\ell'(q)} \geq 1$ for all $q \neq 0$.[8] In addition, ℓ rises sufficiently slowly to make expected utility well-defined. η and ω are random variables - in the example, the tastes of consumers that are served by the two divisions - whose realizations describe the ideal policies from the sender's and the receiver's point of view. These ideal policies are given by $x^R(\omega) = \omega$ and $x^S(\eta) = \eta$, respectively. The realizations of ω and η are unknown at the outset. The designer is interested in joint surplus[9]

$$u^H(x,\eta,\omega) = -\ell(x-\eta) - \ell(x-\omega).$$

Note that the the loss function $\ell(\cdot)$ is the same for both divisions. In Section 5, we extend the analysis to heterogeneous losses.

The decision process is organized as follows. The sender gets to observe noisy signals

$$s_\omega = \omega + \varepsilon_\omega \quad \text{and} \quad s_\eta = \eta + \varepsilon_\eta,$$

where ε_ω and ε_η are uncorrelated noise terms. The receiver is in charge of making the decision. The designer shapes the communication between the sender and the receiver by controlling the research that the sender conducts. Formally, the designer chooses the amount of noise in the sender's signals, that is the variances $\sigma^2_{\varepsilon_\omega}$ and $\sigma^2_{\varepsilon_\eta}$ of the noise terms ε_ω and ε_η. This choice is publicly observable. However, the realizations of signals s_ω and s_η are privately observed by the sender. The sender communicates with the receiver, who finally chooses x. There is no cost of sending messages and the receiver is unable to commit to the

[8] Examples include $\ell(q) = q^{2n}$ for $n \in \mathbb{N}$.

[9] As shown by Alonso et al. (2008), profit sharing between headquarters and the divisions in a firm gives rise to such headquarters preferences.

action x as a function of the information he receives, so communication is modeled as cheap talk in the sense of Crawford and Sobel (1982).

To make the updating about the underlying states tractable we place restrictions on the joint distribution of $\omega, \eta, \varepsilon_\omega$ and ε_η. We focus on an environment where conditional means are linear functions of the observed information. Moreover, linear transformations of the underlying random variables follow the same class of distribution as the underlying random variables do. As is well known, these assumptions are satisfied, e.g., if $\omega, \eta, \varepsilon_\omega$ and ε_η are jointly normally distributed. However, these assumptions are generally fulfilled by all members of the class of elliptical distributions, which includes the Normal distribution as a special case. In what follows, we term the joint distribution of ω, η, s_ω and s_η the *information structure*. An information structure is feasible if it belongs to the elliptical class, has a density function, finite first and second moments, and if the marginal joint distribution of ω and η equals the prior distribution. Given these assumptions, the joint density of a random vector $\mathbf{Y}$ of dimension n can be written as $f_{\mathbf{Y}}(\mathbf{y}) = \kappa_n |\mathbf{\Sigma}|^{-\frac{1}{2}} \phi\left((\mathbf{y}-\mu)' \mathbf{\Sigma}^{-1}(\mathbf{y}-\mu)\right)$, where μ is the mean vector, $\mathbf{\Sigma}$ is up to a constant factor equal to the covariance matrix, $\phi(\cdot)$ is a given function, and κ_n a scale factor, which we simply denote $\kappa = \kappa_1$ in the one-dimensional case.[10]

We assume that all the differences in preferences are unsystematic and random. Formally, we assume that $\mathbb{E}[\omega] = \mathbb{E}[\eta]$. This amounts to saying that systematic differences in preferences - where the sender wishes to push the decision in a particular direction relative to the receiver's preferred choice - have been eliminated prior to the current interaction. We show later that a simple way to achieve this is by means of a linear transfer price scheme between the sender and the receiver; for the time being, we simply work from this situation as an assumption. This does not imply that preferences are aligned. It only implies that based on prior information no differences of opinions are expected. In addition, we impose the innocuous normalization that $\mathbb{E}[\omega] = \mathbb{E}[\eta] = \mathbb{E}[\varepsilon_\omega] = \mathbb{E}[\varepsilon_\eta] = 0$. The covariance matrix is described by $\sigma_\omega^2 \equiv Var(\omega)$, $\sigma_\eta^2 \equiv Var(\eta)$, $\sigma_{\varepsilon_i}^2 \equiv Var(\varepsilon_i)$ for $i = \omega, \eta$, and $\sigma_{\omega\eta} \equiv Cov(\omega, \eta)$. The covariances involving the noise terms are zero by assumption. The

[10] The Normal distribution corresponds to the case $\phi(u) = e^{-\frac{u}{2}}$ and $\mathbf{\Sigma}$ identically equal to the covariance matrix. The factor κ_n depends on n to make f a density. Other members of the elliptical class include, e.g., the exponential power distribution (and as a special case the Laplace) or the logistic distribution. For more details on elliptical distributions see, e.g., Fang et al. (1990).

coefficient of correlation between ω and η is defined as

$$\rho \equiv \frac{\sigma_{\omega\eta}}{\sigma_\omega \sigma_\eta}.$$

To complete the description of the model, consider the ideal policies from the sender's and the receiver's perspective if each of them had access to the information s_ω and s_η.

Lemma 1 *As functions of the underlying signal realizations, s_ω, s_η, the ideal choice functions of the receiver and the sender are*

$$x^R(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^R(x,\omega)\middle| s_\omega, s_\eta\right] = \mathbb{E}\left[\omega| s_\omega, s_\eta\right] = \alpha^R s_\omega + \beta^R s_\eta$$

and

$$x^S(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^S(x,\eta)\middle| s_\omega, s_\eta\right] = \mathbb{E}\left[\eta| s_\omega, s_\eta\right] = \alpha^S s_\omega + \beta^S s_\eta,$$

where α^i, β^i for $i = R, S$ are weights, independent of s_ω, s_η.
Unless $\sigma_\omega^2 = \sigma_\eta^2 = \sigma_{\omega\eta}$, $x^R(s_\omega, s_\eta) \neq x^S(s_\omega, s_\eta)$ for all $s_\omega, s_\eta \neq 0$.

The optimal choice functions correspond to the conditional expectations and conditional expectations are linear in our statistical framework. The intuition is familiar from the Normal distribution-quadratic loss case; we state the result as a lemma, because we prove the generalization both with respect to a wider class of distributions and loss functions.

The sender and the receiver disagree on the optimal course of action for almost all signal realizations unless their ideal policies are perfectly correlated with identical marginal distributions, in which case they are essentially identical. The coefficient of correlation captures the alignment of interests in an intuitive way. It is easy to show that no meaningful communication is possible if $\rho \leq 0$.[11] To focus on the interesting case, we assume that $0 < \rho < 1$.

It is worth pausing for a minute to discuss the crucial assumptions and differences to other approaches in the literature. The main difference is the way we capture conflicts of interests. We assume identical loss functions for sender and receiver and capture all the differences between them by the random variables ω and η and their distributions. The first moments describe ideal policies, the second moments shape expected utilities. Assuming equal prior expectations amounts to saying that differences of opinion prior to the current interaction

[11] A formal proof of this statement is available from the authors upon request.

have been eliminated. The remaining conflicts are random and unsystematic, in the sense that their expected value is zero. We make these assumptions, because it is by now well known that communication does not work well with systematic differences of opinions. In contrast, it is not yet known how well communication can work with unsystematic differences of opinions.

We analyze the game proceeding backwards, starting with the inference that the sender draws from observing facts and the ensuing communication continuation games. We then reduce the model to one where communication is about inferences instead of facts and discuss the receiver's inferences drawn from the sender's inference. Building on this analysis, we discuss the optimal organizational response to filtering information this way, the optimal amount and kind of information that the organization acquires.

3 The sender as a strategic information channel

Suppose that the designer has chosen a research policy - formally, an information structure - and the sender gets to observe the results of the research. What part of the observed information is the sender willing to share with the receiver at all?

3.1 Limits to communication

We focus on Bayesian equilibria in the communication game. After observing signal realizations s_ω, s_η, the sender sends a message $m \in \mathbb{M}$ to the receiver. The message space is sufficiently rich; we do not impose any restrictions on $\mathbb{M}$. A pure sender strategy maps the sender's information into messages $M : \mathbb{R}^2 \to \mathbb{M}$, $(s_\omega, s_\eta) \mapsto m$. A mixed strategy is a probability distribution over pure strategies. It is enough to consider pure message strategies for the sender.[12] A pure receiver strategy maps messages into actions, $X : \mathbb{M} \to \mathbb{R}$, $m \mapsto x$. The receiver updates his belief about the sender's type after observing the sender's message and acts optimally against this belief. Mixing is never optimal for the receiver. The following lemma derives an upper bound on the information that can be communicated in

[12]More specifically, it is standard in the literature to look at the most informative equilibria and these equilibria involve pure strategies in our game. Therefore, we abstain from introducing the notational clutter to deal formally with mixed strategies.

any equilibrium of the communication game. In particular, the sender is willing to share his inference but not the underlying facts.

Lemma 2 *Any equilibrium is essentially equivalent to one where the sender's message strategy is a function of $\alpha^S s_\omega + \beta^S s_\eta$ only and all sender types s_ω, s_η such that $\alpha^S s_\omega + \beta^S s_\eta =$ constant induce the same action.*

Define the statistic

$$\theta \equiv \alpha^S s_\omega + \beta^S s_\eta.$$

All sender types with signal realizations s_ω, s_η adding up to θ share the same ideal policy, θ. Moreover, with symmetric loss functions, the sender's preferences over distinct actions depend only on the distance of these actions to θ. Hence, the set of types who share the same θ induce at most two distinct actions, and these actions need to be equidistant from θ in any equilibrium. We show in the proof that this implies that types who share the same θ induce the same action for almost all θ. Subsets of types with one out of a countable number of realizations of θ may induce distinct actions even though they share the same θ. However, since these types have measure zero and pool with a set of types of positive measure that induce single actions only, the strategies of types with the same θ that induce distinct actions do not influence the receiver's beliefs nor her optimal actions. Hence, for any equilibrium, where some types with the same θ play distinct strategies, we can find an equilibrium with the same induced receiver actions and the same expected payoffs, where all sender types with the same θ induce the same action. The two equilibria are only essentially equivalent in that they involve distinct sender strategies on measure zero sets. However, for all that matters economically, they are the same. Obviously, the lemma also implies that it is impossible to elicit the information s_ω, s_η from the sender, unless the ideal policies of sender and receiver coincide altogether.

Corollary 1 *Truthful communication of the underlying information, s_ω, s_η, is an equilibrium if and only if $\sigma_\omega^2 = \sigma_\eta^2 = \sigma_{\omega\eta}$.*

Since induced actions depend only on the realization of θ, the sender is willing to reveal at most the inference he draws from the facts, that is θ, but never the underlying facts. Hence, we can characterize any equilibrium of the communication game in terms of communication about the sender's inference, θ, only.

3.2 Inference from inference and conflicts

From the ex ante perspective, before the signals are realized, the sender's inference is random itself. Any given choice of information structure gives rise to a joint distribution of ω, η, and θ. Given that $\omega, \eta, \varepsilon_\omega$ and ε_η follow a joint elliptical distribution, the random variables ω, η, and θ follow a joint elliptical distribution as well. One can show that the moments involving θ are given by $\mathbb{E}[\theta] = 0$ as well as

$$Var(\theta) = \sigma_\eta^2 \frac{\frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2} + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\rho^2 + 1 - \rho^2}{\left(1 + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}\right)\left(1 + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\right) - \rho^2}, \tag{1}$$

$$Cov(\omega, \theta) = \sigma_{\omega\eta} \frac{\frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2} + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2} + 1 - \rho^2}{\left(1 + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}\right)\left(1 + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\right) - \rho^2}, \tag{2}$$

and

$$Cov(\eta, \theta) = Var(\theta). \tag{3}$$

Equations (1) and (2) depend crucially on the normalized noise variances, $\frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}$ and $\frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}$. By construction of θ, only covariance matrices with $Cov(\eta, \theta) = Var(\theta)$ are possible. For all that matters in terms of induced choices and payoffs, we can analyze our model in terms of this reduced form joint distribution of inference and underlying states.

What inference would the receiver draw if the sender communicated his inference? Since the joint distribution of ω, η, and θ is firmly within the class that has linear conditional means, the receiver's ideal policy conditional on observing θ is

$$\mathbb{E}[\omega|\,\theta] = \frac{Cov(\omega, \theta)}{Var(\theta)} \cdot \theta. \tag{4}$$

The conditional expectation corresponds to the linear regression of the unknown state on the observed information. To understand the slope of the regression, note that the regression of η on θ is simply

$$\mathbb{E}[\eta|\,\theta] = \frac{Cov(\eta, \theta)}{Var(\theta)} \cdot \theta = \theta. \tag{5}$$

Clearly, given that θ is the conditional expectation of η given the underlying facts, the sender does not revise his conditional expectation if shown θ again. In contrast, the receiver's inference corrects for the relative informational content of the sender's inference, θ, with respect to the underlying states ω and η: by Equation (3), the slope $\frac{Cov(\omega,\theta)}{Var(\theta)}$ corresponds to $\frac{Cov(\omega,\theta)}{Cov(\eta,\theta)}$. If the sender gets to observe information that is relatively more informative about ω than about η, then $Cov\,(\omega,\theta) > Cov\,(\eta,\theta)$ and the receiver's ideal policy attaches a higher weight to the information θ than the sender's ideal policy. The situation is reversed if the sender gets to see information that is relatively more useful to the sender. The regressions have identical slopes if the sender's inference is equally informative about ω and η.

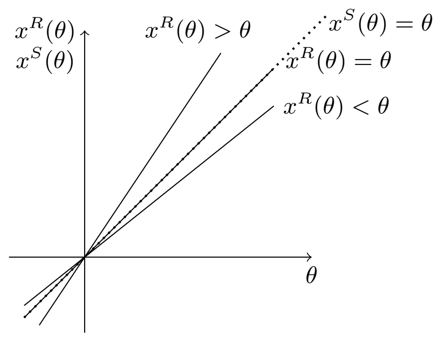

Figure 1: Conflicts with respect to θ between sender and receiver.

The difference $\theta - \mathbb{E}\,[\omega|\,\theta]$ describes the bias of the sender relative to the receiver. If the sender observes information that is relatively more informative about η, then the sender has incentives to exaggerate. If the sender's information is relatively more informative about ω, then the sender has incentives to downplay. Finally, there is no bias when communicating about the sender's inference when the sender's inference θ is equally informative about ω and η. For convenience, the three cases are depicted in Figure 1.

4 Optimal information structures

We now address the designer's problem of choosing an optimal information structure. What information should the sender get to observe about each of the underlying taste parameters, ω and η? The sender's information impacts on payoffs through two channels. Firstly, assuming honest transmission of the sender's inference, the relative informational content of θ impacts directly on the sender's and the receiver's expected payoff from making a receiver-optimal decision based on θ. Secondly, the relative informational content determines the sender's bias in the communication game and thus impacts on the amount of information that is transmitted through communication. It is helpful to look at the two margins separately. Therefore, we begin our analysis with the clearly unrealistic case where the sender's inference θ becomes publicly available.[13] In a second step, in Section 4.2, we look at the case of main interest, where θ is private information.

To streamline the exposition, we present our analysis first assuming that marginals are identical. That is, we assume $\sigma_\omega^2 = \sigma_\eta^2$. We discuss the role of this assumption and abandon it in Section 5.2.2 below.

4.1 Public inferences

4.1.1 The designer's problem

If the receiver observes the sender's inference θ, then she follows the policy $x^R(\theta) = \mathbb{E}[\omega|\,\theta] = \frac{Cov(\omega,\theta)}{Var(\theta)} \cdot \theta$, resulting in a loss of $\ell\left(\frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \omega\right)$ for the receiver and a loss of $\ell\left(\frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \eta\right)$ for the sender. Both losses depend only on sums of the underlying random variables, $\zeta \equiv \frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \omega$ and $\tau \equiv \frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \eta$, which are again elliptical. Let σ_ζ^2 and σ_τ^2 denote the variances of ζ and τ and let $z \equiv \frac{\zeta}{\sigma_\zeta}$ and $t \equiv \frac{\tau}{\sigma_\tau}$ denote the standardized arguments of the loss functions. As demonstrated formally in the Appendix, we can write the designer's problem as

$$\max_{Cov(\omega,\theta),Var(\theta)} -\int \ell(\sigma_\zeta z)\,\kappa\phi(z)\,dz - \int \ell(\sigma_\tau t)\,\kappa\phi(t)\,dt$$
$$s.t.\ Cov(\omega,\theta), Var(\theta) \text{ feasible.}$$

where z and t follow a spherical distribution with density $\kappa\phi(\cdot)$.[14]

The sender's and the receiver's expected utilities depend negatively on a residual variance that measures the residual uncertainty after using θ optimally from the receiver's perspective.

[13] We can think of this as some form of mediated information transmission; the sender's information s_ω, s_η is aggregated to $\alpha^S s_\omega + \beta^S s_\eta = \theta$ and then mechanically transmitted to the receiver.

[14] In the Gaussian case, the spherical distribution corresponds to the Standard Normal.

Naturally, the residual uncertainty for the receiver is

$$\sigma_\zeta^2 = \sigma_\omega^2 - \frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)} = Var\left(\omega|\,\theta\right), \tag{6}$$

where the second equality holds because θ is used optimally from the receiver's perspective.[15] In contrast, θ is in general not used optimally from the sender's perspective. The residual uncertainty that the sender faces when θ is used according to the policy $x^R(\theta)$ is

$$\sigma_\tau^2 = \sigma_\eta^2 - \left(2Cov\left(\omega,\theta\right) - \frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)}\right), \tag{7}$$

which differs from $Var\left(\eta|\,\theta\right) = \sigma_\eta^2 - Var\left(\theta\right)$ unless (5) and (4) are identically equal to each other.

Consider now the feasible set of information structures. Not any joint distribution of ω, η, θ is a feasible reduced form information structure, because θ must be derived from Bayesian updating by the sender about η, conditioning on the information that the sender gets to see. Thus, a joint distribution of ω, η and θ is feasible only if there are noise variances $\sigma_{\varepsilon_\omega}^2$ and $\sigma_{\varepsilon_\eta}^2$ that, together with the prior distribution, induce the joint distribution. The following lemma makes the restrictions from Bayesian updating explicit.

Lemma 3 *A joint distribution of ω, η, θ can be generated through Bayesian updating if and only if $Cov\left(\omega,\theta\right) \in \left[0, \sigma_{\omega\eta}\right]$ and for any given $Cov\left(\omega,\theta\right) = C$, $Var\left(\theta\right) \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho C, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C\right]$.*

$Var\left(\theta\right)$ and $Cov\left(\omega,\theta\right)$ are jointly constrained to lie in the triangle described in Figure 2. We call the feasible set Γ. To understand the shape of Γ, note that any pair of normalized noise variances, $\frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}, \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2} \geq 0$, results in a $Cov\left(\omega,\theta\right) \leq \sigma_{\omega\eta}$. The covariance is maximal if at least one of the signals is perfectly precise. In the limiting case of infinitely noisy signals, the sender does not revise his prior at all and so both $Var\left(\theta\right)$ and $Cov\left(\omega,\theta\right)$ are zero. If the sender observes a signal s_η without noise, $\sigma_{\varepsilon_\eta}^2 = 0$, then his posterior mean becomes identically equal to η and the resulting variance is $Var\left(\theta\right) = \sigma_\eta^2$. If the sender observes s_ω without noise, $\sigma_{\varepsilon_\omega}^2 = 0$, and the signal s_η is infinitely noisy, $\sigma_{\varepsilon_\eta}^2 \to \infty$, then $Var\left(\theta\right) = \rho^2\sigma_\eta^2$, because the sender's posterior mean rises less than one for one with the sender's observation.

[15] For the derivation of the conditional second moments see Lemma A.1 in the Appendix.

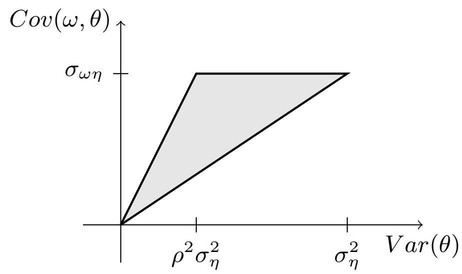

Figure 2: The feasible set of information structures, Γ.

By continuity, any pair of covariance and variance in the interior of the triangle can be generated by some pair of noise variances. Finally, Γ is always nonempty, because the lowest feasible $Var(\theta)$ for any given $Cov(\omega,\theta)$ is below the highest feasible $Var(\theta)$ by the Cauchy-Schwarz inequality, $\sigma_{\omega\eta}^2 \leq \sigma_\eta^2\sigma_\omega^2$.[16]

4.1.2 Equalizing residual uncertainty

We can now restate the designer's problem as

$$\max_{Cov(\omega,\theta),Var(\theta)} -\int \ell\left(\sigma_\zeta z\right)\kappa\phi\left(z\right)dz - \int \ell\left(\sigma_\tau t\right)\kappa\phi\left(t\right)dt$$
$$s.t.\ Cov\left(\omega,\theta\right), Var\left(\theta\right) \in \Gamma,$$

where σ_ζ and σ_τ are defined in (6) and (7). The designer maximizes a continuous objective function on a compact domain, so the problem is well defined and a solution exists. The solution takes the following form:

Theorem 1 *Suppose that the sender and the receiver are equally uncertain ex ante, $\sigma_\omega^2 = \sigma_\eta^2$. If the loss function satisfies $\frac{q\ell''(q)}{\ell'(q)} > 1$ for all $q \neq 0$, then the designer's problem of choosing an optimal information structure has a unique solution, which is given by $Var(\theta)^* =$*

[16]We include edges and vertices in the feasible set that result from taking limits. The limiting posterior distributions and moments when one noise variance goes out of bounds converge to the distribution when only one signal is received; the limiting case when both noise variances go out of bounds converges to the distribution when no signal at all is received, the prior.

$Cov(\omega,\theta)^* = \sigma_{\omega\eta}$. If the loss function satisfies $\frac{q\ell''(q)}{\ell'(q)} = 1$ for all $q \neq 0$ (corresponding to the quadratic case), then any information structure satisfying $Cov(\omega,\theta) = \sigma_{\omega\eta}$ is optimal.

We solve the problem by maximizing sequentially with respect to $Var(\theta)$ and $Cov(\omega,\theta)$. For a given level of $Cov(\omega,\theta)$, the designer's problem resembles a risk sharing problem. The sender and the receiver both dislike higher residual uncertainty and an increase of $Var(\theta)$ increases (6), the residual uncertainty the receiver faces, and decreases (7), the residual uncertainty the sender faces. For a sufficiently convex loss function, the problem is single-peaked in $Var(\theta)$ and has a unique maximum at the point where the residual uncertainty for the sender and the receiver is equalized. Equating (6) and (7) and solving for $Var(\theta)$, we obtain

$$Var(\theta)^* = Cov(\omega,\theta).$$

The residual uncertainty for each party is then equal to the residual uncertainty that the receiver faces, $Var(\omega|\theta) = \sigma_\omega^2 - Cov(\omega,\theta)$. Since this is a decreasing function of $Cov(\omega,\theta)$, it is optimal to choose $Cov(\omega,\theta)$ as high as possible,

$$Cov(\omega,\theta)^* = \sigma_{\omega\eta}.$$

The unique optimum corresponds to the intersection of the dashed and the solid line in Figure 3. The role of the curvature condition is to guarantee uniqueness of the optimal $Var(\theta)$. For the quadratic loss function, the joint payoff becomes linear in the residual variances, which implies that the receiver's loss from increasing $Var(\theta)$ just offsets the sender's gain and thus the sum of their payoffs becomes independent of $Var(\theta)$. Hence, any information structure with the highest feasible $Cov(\omega,\theta)$, depicted as the solid line in the figure, is optimal.

The optimum can be understood by decomposing information into its common and idiosyncratic content. Since $Cov(\omega,\eta|\theta) = \sigma_{\omega\eta} - Cov(\omega,\theta)$, $Cov(\omega,\theta)$ measures the amount of common information. Naturally, the optimal information structure contains all the common information there is,

$$Cov(\omega,\eta|\theta)^* = \sigma_{\omega\eta} - Cov(\omega,\theta)^* = 0,$$

implying that conditional on θ, the taste parameters become uncorrelated. $Var(\theta)$ measures the amount of idiosyncratic information. Since there is only one signal, θ, idiosyncratic infor-

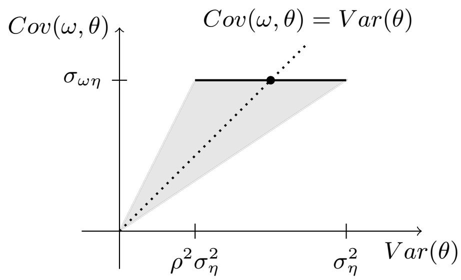

Figure 3: The optimal information structure maximizes $Cov(\omega, \theta)$. For sufficiently convex loss functions it is unique and satisfies $Cov(\omega, \theta) = Var(\theta)$.

mation necessarily involves a trade-off: $Var(\omega|\theta)$ is increasing in $Var(\theta)$, while $Var(\eta|\theta)$ is decreasing in $Var(\theta)$.

In terms of the underlying signals, the designer allows the sender to observe ω without noise, $\sigma^2_{\varepsilon_\omega} = 0$, but adds noise $\sigma^2_{\varepsilon_\eta} = \frac{1-\rho^2}{\rho}\sigma^2_\eta$ to the signal about η. If the signal s_η were perfectly precise, then the sender would not pay any attention to the signal s_ω. While θ would still contain the maximum amount of common information, θ would not be informative enough about ω and so the receiver would face too much residual uncertainty. Hence, noise is needed to keep the sender from using the signal that is of primary importance to him exclusively.

4.2 Private inferences

We now consider the case of main interest where the sender has private information about θ and thus is free to make up any statement he likes. As is standard in the literature, we assume that the sender and the receiver are able to coordinate on the ex ante Pareto optimal equilibrium in the communication game. The optimal information structure eliminates conflicts in a certain, well defined sense:

Theorem 2 *Let the sender and the receiver face equal prior uncertainty, $\sigma^2_\omega = \sigma^2_\eta$. Then, the unique optimal information structure chosen by the designer satisfies $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$. The Pareto best equilibrium of the ensuing continuation game involves smooth strategies; the sender truthfully announces θ, $m^*(\theta) = \theta$ for all θ, and the receiver takes the sender's* *advice at face value,* $x^*(m) = m$ *for all* m*. All parties' payoffs are the same as if the sender were given the right to choose the action* x *directly.*

The theorem is a straightforward implication of our preceding results in conjunction with a verification that the described strategies constitute an equilibrium of the communication game. Since the designer cannot improve upon his payoff compared to the case where θ is public information, the situation corresponds to an optimum if this payoff is reached. Suppose the receiver believes that the sender plays the message strategy $m(\theta) = \theta$ for all θ. Then, his best reply is the action strategy $x^*(m) = \frac{Cov(\omega,\theta)^*}{Var(\theta)^*} \cdot m = m$ for all m. The sender, who anticipates this policy, induces his ideal policy by being truthful about θ, so the construction is indeed an equilibrium. Note that in this equilibrium the strategies of both players are smooth - in fact, linear - functions.

Since $x^*(m^*(\theta)) = \theta$ for all θ, the sender's optimal policy is implemented for all θ. Consequently, whether the sender communicates with the receiver or whether the sender is given the right to choose the policy, the payoffs of all parties involved are exactly the same.[17] The intuition is that, for equal marginals, an information structure that equalizes residual uncertainty automatically eliminates any bias in the use of information. Formally, $Cov(\omega, \theta)^* = Var(\theta)^*$ implies

$$x^R(\theta) - x^S(\theta) = \left(\frac{Cov(\omega,\theta)^*}{Var(\theta)^*} - 1\right) \cdot \theta = 0 \quad \forall \theta.$$

Note that there remains a conflict between sender and receiver with respect to using the underlying signals, s_ω and s_η. However, the receiver simply cannot do better than follow the sender's advice, because based on observing the sender's inference θ, a garbled piece of information, the receiver's ideal choice coincides with the sender's ideal choice based on observing the underlying signals. The sender is willing to share his inference despite disagreement too. The sender knows that the receiver would ideally like to choose an action that matches the state ω, not θ. However, under the optimal information structure, the sender's recommendation θ and the difference $\omega - \theta$ become uncorrelated. Put differently,

[17] Note that the problem of multiple solutions for the quadratic loss case if θ is public information is eliminated, because truthful communication now requires that $\frac{Cov(\omega,\theta)^*}{Var(\theta)^*} = 1$.

the optimal information structure *orthogonalizes* the conflict between the sender and the receiver and the recommendation and hence removes any impediments to communication.[18]

Communication is in fact unsurpassed by any form of delegation, even *optimal delegation*. Even if the designer or the receiver had the right to constrain the sender's discretion under delegation, they would not want to make use of this right. The sender's optimal choice is necessarily a function of his inference θ only, and the sender uses this inference in the receiver's best interest. Hence, constraining the sender's discretion under delegation decreases the receiver's payoff and joint surplus.

4.3 The quality of decision making

Under the optimal information structure information is lost because only inferences are transmitted. How much is lost by such garbling and how does this depend on the underlying conflicts?

We can measure the amount of information transmitted in equilibrium by the variance of induced choices; the higher this variance, the more information is transmitted. The designer throws in just enough noise to ensure that $Var(\theta) = \sigma_{\omega\eta}$. For identical priors, the variance of the induced choice is thus

$$Var(\theta) = \rho\sigma_{\eta}^{2}.$$

The higher is ρ, the more variable the induced choice. In the limit as $\rho \to 1$, the sender truthfully announces η and the variance of choices approaches σ_{η}^{2}. There are two reasons why increasing ρ results in an improvement of information transmission. Recall that the sender always observes ω without noise. The higher is ρ, the higher the attention the sender pays to this signal and the more this signal is reflected in the sender's preferred choice. Moreover, the sender observes η with an amount of noise equal to $\sigma_{\varepsilon_\eta}^{2} = \frac{1-\rho^2}{\rho}\sigma_{\eta}^{2}$, a decreasing function of ρ. The higher is ρ, the more precise the sender's signal about the sender-relevant random

[18]Note that our construction is very different from the orthogonal decomposition in Battaglini (2002). Closer in spirit is an observation by Li and Madarász (2008), who remark in an analysis of mandatory disclosure of biases, that communication works well if the bias is independently and symmetrically distributed. In contrast to that work, the bias is endogenous in our work. For further analyses of privately known unidirectional biases, see Morgan and Stocken (2003) and Dimitrakas and Sarafidis (2005).

variable η. So, senders with better aligned interests are more trustworthy to begin with and get endowed with more precise information, rendering their advice even more valuable.[19]

5 Asymmetric environments

Our analysis so far seemingly relies quite heavily on symmetry assumptions. In particular, our main results are valid for equal prior means, equal prior variances, and identical loss functions. We now show that these assumptions are inessential if one is willing to go back to quadratic loss functions, the canonical case studied in the literature. We maintain this assumption throughout the following section.

5.1 Unequal prior expectations

Suppose that $\mathbb{E}[\omega] = 0 \neq \mathbb{E}[\eta]$. Suppose further that the sender pays $p \cdot x$ to the receiver when the action x is chosen. The payment can become negative, depending on the sign of x. The price p is set at $\frac{\mathbb{E}[\eta]-\mathbb{E}[\omega]}{2} = \frac{\mathbb{E}[\eta]}{2}$. It is easy to see that the sender's and the receiver's ideal action based on prior information alone is now equal to $x = \frac{\mathbb{E}[\eta]}{2}$. Let $\hat{\omega}$ and $\hat{\eta}$ denote the random bliss points of the receiver and the sender in the new situation. Moreover, suppose the sender observes signals $s_{\hat{\omega}} = \hat{\omega} + \varepsilon_{\hat{\omega}}$ and $s_{\hat{\eta}} = \hat{\eta} + \varepsilon_{\hat{\eta}}$ and let $\hat{\theta}$ denote the sender's conditional expectation in the new situation. $\hat{\theta}$ equals now the prior mean, $\frac{\mathbb{E}[\eta]}{2}$ plus α^S times the surprise element in $s_{\hat{\omega}}$ plus β^S times the surprise element in $s_{\hat{\eta}}$. The random variables $\left(\hat{\omega}, \hat{\eta}, \hat{\theta}\right)$ now have mean vector $\left(\frac{\mathbb{E}[\eta]}{2}, \frac{\mathbb{E}[\eta]}{2}, \frac{\mathbb{E}[\eta]}{2}\right)$. The constant shift does not affect the covariance matrix. Hence, our entire analysis goes through as is.

Transfer prices are commonly used in practice. They are easy to administer in the present situation, because the price p only depends on information that is available before the current interaction takes place. However, even if this is allowed for, our analysis shows that conflicts may still arise due to information that arrives after transfer prices have been set and shows how such obstacles can be overcome.[20]

[19] In the limit, the feasible set of information structures converges to the 45° line and any piece of information is equally informative about ω and η. Hence endowing the sender with perfect information becomes optimal. At the same time the underlying interests of sender and receiver become perfectly correlated.

[20] Note that the payment scheme is restricted in many ways: it takes the form of an indirect scheme that

For the remainder of our analysis, we revert to the case of equal prior expectations.

5.2 Heterogeneous losses

Expected losses for the sender and the receiver can differ for two reasons: their priors can have different variances or they may care about the decision to a different extent. We now develop the common framework to discuss both issues.

5.2.1 Public inferences in asymmetric environments

Suppose that the the sender and the receiver face unequal prior uncertainty, $\sigma_\omega^2 \neq \sigma_\eta^2$. In addition, they differ with respect to the relative value they attach to reducing prior uncertainty. In particular, suppose that $u^S(x,\eta) = -\gamma(x-\eta)^2$ and $u^R(x,\omega) = -(1-\gamma)(x-\omega)^2$ for some $\gamma \in [0,1]$. Under public inferences, the designer's problem is now

$$\max_{Cov(\omega,\theta),Var(\theta)} -(1-\gamma)\left(\sigma_\omega^2 - \frac{Cov(\omega,\theta)^2}{Var(\theta)}\right) - \gamma\left(\sigma_\eta^2 - 2Cov(\omega,\theta) + \frac{Cov(\omega,\theta)^2}{Var(\theta)}\right)$$
$$s.t.\ Cov(\omega,\theta), Var(\theta) \in \Gamma.$$

Recall from Theorem 1 that the solution is $Cov(\omega,\theta) = \sigma_{\omega\eta}$ together with any feasible $Var(\theta)$ for the case $\gamma = \frac{1}{2}$. It is easy to prove that the solution is $Cov(\omega,\theta) = \sigma_{\omega\eta}$ and $Var(\theta) = \rho^2\sigma_\eta^2$ for the case $\gamma < \frac{1}{2}$. Since the receiver's utility enters the designer's objective with a relatively higher weight, the most informative information structure from the receiver's perspective is chosen at the optimum. For the case $\gamma > \frac{1}{2}$, two cases can arise. If $\sigma_\eta^2 \geq \frac{1}{2}\sigma_{\omega\eta}$, then the optimal information structure from the sender's perspective, $Cov(\omega,\theta) = \sigma_{\omega\eta}$ and $Var(\theta) = \sigma_\eta^2$, is optimal. If $\sigma_\eta^2 < \frac{1}{2}\sigma_{\omega\eta}$, then the sender's preferred information structure is optimal for sufficiently low γ. Beyond that, it becomes optimal not to acquire any information. The reason is as follows. For high values of γ, the designer wishes to choose an information structure that is primarily useful to the sender. However, it is the receiver who actually takes actions. If $\sigma_\eta^2 < \frac{1}{2}\sigma_{\omega\eta}$, then the receiver overreacts by a factor larger than

depends on the action and its functional form is very restricted. For an analysis of optimal money payments, with commitment to such payments, see Krishna and Morgan (2008).

two to θ relative to the sender. This means that the action $x = 0$ is uniformly closer to the sender's preferred policy, $x^S(\theta) = \theta$, than $x^R(\theta) = \frac{\sigma_{\omega\eta}}{\sigma_\eta^2} \cdot \theta$ is.

The solution to the designer's problem under public inferences features generically $Cov(\omega, \theta)^* \neq Var(\theta)^*$ in asymmetric environments. One is tempted to conclude that the same must be true for the private inference environment. However, this conclusion is flawed.

From the perspective of the robustness of our main analysis, two questions must be addressed: feasibility and optimality. It could be that, even though a solution featuring $Cov(\omega, \theta)^* = Var(\theta)^*$ would be desirable, it is no longer feasible, due to asymmetric priors. Or, even though feasible, an information structure enabling smooth communication could no longer be desirable given a high enough weight attached to the utility of one particular party.

We now address both issues in sequence. Since the analysis of unequal utility weights requires more intense algebra, we address feasibility first for the case $\gamma = \frac{1}{2}$.

5.2.2 Private inferences and unequal prior variances

If sender and receiver face unequal prior uncertainty, $\sigma_\omega^2 \neq \sigma_\eta^2$, but the designer attaches equal weights to each party's payoff, $\gamma = \frac{1}{2}$, then acquiring balanced information remains optimal as long as it is feasible. Formally, we have the following result:

Proposition 1 *Assume quadratic loss functions and that the designer attaches equal weights to each party's payoff. Moreover, suppose that* $\min\{\sigma_\omega^2, \sigma_\eta^2\} \geq \sigma_{\omega\eta}$. *Then, the designer's optimal choice of information structure is unique and given by* $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$. *All parties receive the same expected payoff, regardless of who has the right to choose the action* x.

Recall that by Theorem 1 any information structure satisfying $Cov(\omega, \theta) = \sigma_{\omega\eta}$ is optimal for quadratic losses if θ is public. Hence, to show that the designer can reach the same expected payoff under communication of unverifiable information - and under delegation - it suffices to show that the admissible set of information structures contains the element $Var(\theta) = Cov(\omega, \theta) = \sigma_{\omega\eta}$. The condition in the proposition is equivalent to

$$\frac{\sigma_\eta}{\sigma_\omega}\rho \leq 1 \leq \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho},$$

which guarantees that the 45° line is an element of the feasible set, Γ. We need to rule out very asymmetric priors where $\sigma_\omega^2 > \sigma_{\omega\eta} > \sigma_\eta^2$ or $\sigma_\eta^2 > \sigma_{\omega\eta} > \sigma_\omega^2$ that would render the solution $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$ infeasible.[21]

Note that nonverifiability makes the solution unique. While the sum of residual variances is constant for all information structures with the highest feasible $Cov(\omega, \theta)$, there is only one information structure among them that makes the signal θ equally useful for the sender and the receiver and thus ensures that truthful communication about θ is an equilibrium.

As a converse to the proposition, with very asymmetric prior distributions, it is not feasible to make the sender's conditional expectation equally useful to both parties. In that case, even though desirable, it is not possible to acquire balanced information. Hence, no equilibrium featuring truthful communication of θ can exist and delegation and communication are no longer outcome equivalent.

5.2.3 Private inferences and heterogeneous losses

Suppose that the prior distributions are not too different, in the sense that $\min\{\sigma_\omega^2, \sigma_\eta^2\} \geq \sigma_{\omega\eta}$, and let the sender and the receiver be heterogeneous in the intensity of their losses, $\gamma \neq \frac{1}{2}$. We know that acquiring balanced information is not an optimum of the designer's problem under public inferences under these assumptions. Throughout the previous analysis, the method of proof was to show that the optimum under public inferences is also attainable under private inferences. Obviously, a different method is now required. We need to attach precise values to communication also for some information structures that feature $Var(\theta) \neq Cov(\omega, \theta)$ and so make truthful communication of θ impossible. This is a complex task. Fortunately, we can capitalize on a companion paper of ours (Deimen and Szalay (2016)), where we develop the methods to deal with this task. For reasons of space, we refer the reader to our companion paper Deimen and Szalay (2016) for the proofs of the statements in this section.

If the receiver's and the sender's regressions have different slopes, all communication equilibria are essentially equivalent to equilibria where sender types within intervals pool on inducing the same action by the receiver. To solve for the value of communication in such equilibria, we need to restrict the stochastic environment to one particular member of the

[21] By the Cauchy-Schwarz inequality, it is impossible that both prior variances fall short of the covariance.

elliptical class. In particular, we assume in what follows that all random variables follow a multivariate Laplace distribution.[22] Building on our previous analysis, we know that the marginal distribution of θ is a Laplace distribution too with density

$$f(\theta) = \frac{1}{2}\lambda \exp(-\lambda|\theta|)$$

with mean zero and variance $Var(\theta) = \frac{2}{\lambda^2}$. Note that the scale parameter λ is pinned down by $Var(\theta)$. Let $c \equiv \frac{Cov(\omega,\theta)}{Var(\theta)}$ denote the receiver's regression coefficient.

As usual, there is a plethora of equilibria. We follow the view in the literature that the organization manages to coordinate on the best equilibrium, that is, the equilibrium that maximizes each party's ex ante expected payoff. To prove our result, no explicit characterization of this equilibrium is needed for the case $c > 1$. In contrast, the case $c < 1$ requires more attention. In the latter case, the equilibrium is a limit of one in the following, symmetric class. Sender types with a countable number of distinct values of θ – henceforth, with a slight abuse of terminology, simply types – are indifferent between inducing two actions. These marginal sender types are given by $a_0 = 0$, n positive marginal types $a_1, \ldots, a_n$, and n negative marginal types $-a_n, \ldots, -a_1$, for any $n \geq 1$. By symmetry, it suffices to characterize communication equilibria for $\theta \geq 0$. Let $\Theta_i \equiv [a_{i-1}, a_i)$ for $i \in 1, \ldots, n$ and $\Theta_{n+1} \equiv [a_n, \infty)$ denote generic partition elements in $\mathbb{R}^+$. Denote $\mu_i \equiv \mathbb{E}[\theta|\theta \in \Theta_i]$ for $i = 1, \ldots, n$ and $\mu_{n+1} \equiv \mathbb{E}[\theta|\theta \geq a_n]$. Given quadratic losses, the receiver's optimal response to a message by the sender indicating that $\theta \in \Theta_i$ is given by the conditional expectation of ω given $\theta \in \Theta_i$. By the law of iterated expectations and the linearity of the conditional expectation function, we have

$$x(a_{i-1}, a_i) = c \cdot \mu_i \text{ for } i = 1, \ldots, n$$

as well as $x(a_n, \infty_i) = c \cdot \mu_{n+1}$. The situation corresponds to an equilibrium if the marginal types a_i are indifferent between pooling with types $\theta \in \Theta_i$ or $\theta \in \Theta_{i+1}$, i.e.,

$$a_i - c \cdot \mu_i = c \cdot \mu_{i+1} - a_i \qquad \text{for } i = 1, \ldots, n.$$

Expected utilities can be computed analogously to (6) and (7) with μ, the receiver's ex ante random conditional expectation, replacing θ. We obtain

$$\mathbb{E}u^R(c\mu, \omega) = -\mathbb{E}(c\mu - \omega)^2 = c^2\mathbb{E}(\mu)^2 - \sigma_\omega^2$$

[22] See Kotz et al. (2001) for an in-depth treatment of this distribution.

and

$$\mathbb{E}u^S(c\mu,\eta) = -\mathbb{E}(c\mu-\eta)^2 = c(2-c)\,\mathbb{E}(\mu)^2 - \sigma_\eta^2,$$

where the distribution of μ is derived from the equilibrium interval partition and the underlying distribution of θ. To proceed further from here onwards, a closed form expression for $\mathbb{E}(\mu)^2$ is needed. Deimen and Szalay (2016) show that

$$\mathbb{E}(\mu)^2 = \frac{Var(\theta) - c(\mu_1^n)^2}{2-c}, \tag{8}$$

where μ_1^n is the mean taken over the interval closest to the agreement point, $\theta = 0$, and we write μ_1^n to make the dependence of this mean on n explicit. The stunning feature of the Laplace environment is that the value of communication in an equilibrium of the described class depends on the details of the communication equilibrium only through μ_1^n. This is what makes the Laplace environment so useful.

We are now ready to state headquarters' problem:

$$\max_{C,V} -(1-\gamma)\left(\sigma_\omega^2 - c^2\mathbb{E}(\mu)^2\right) - \gamma\left(\sigma_\eta^2 - c(2-c)\,\mathbb{E}(\mu)^2\right)$$
$$s.t. \quad c = \frac{Cov(\omega,\theta)}{Var(\theta)} \quad \text{and} \quad Cov(\omega,\theta), Var(\theta) \in \Gamma.$$

To solve this problem, we need to understand how the choice variables impact on $\mathbb{E}(\mu)^2$. Deimen and Szalay (2016) characterize the properties of the most informative equilibrium as a function of c. For $c \leq 1$, there exist equilibria in the described class for arbitrary values of n and there exists a limit equilibrium with infinitely many induced actions. In this limit, intervals close to the agreement point, $\theta = 0$, get arbitrarily short. In particular, the length of the first interval converges to zero implying that $\lim_{n\to\infty} \mu_1^n = 0$ and hence $\lim_{n\to\infty} \frac{Var(\theta)-c\left(\mu_1^n\right)^2}{2-c} = \frac{Var(\theta)}{2-c}$. On the other hand, for $c > 1$, in any equilibrium, whether of the described form or not, the number of induced actions is finite. Hence, we necessarily have $\mathbb{E}(\mu)^2 < Var(\theta)$ for $c > 1$. These results provide enough structure to prove the following result:

Proposition 2 *In the Laplace-quadratic environment for all payoff weights $\gamma \in \left[\frac{1}{2}, 1\right)$, the information structure $Var(\theta)^* = Cov(\omega,\theta)^* = \sigma_{\omega\eta}$ is uniquely optimal from the designer's perspective. For $\gamma = 1$, the set of optimal information structures is given by $Cov(\omega,\theta)^* = \sigma_{\omega\eta}$ and any $Var(\theta)^* \geq \sigma_{\omega\eta}$.*

Any information structure satisfying $Var(\theta) = Cov(\omega, \theta)$ gives rise to a joint payoff gain equal to $Cov(\omega, \theta)$. For any $Cov(\omega, \theta), Var(\theta)$ such that $c > 1$, we have

$$(1-\gamma) c^2 \mathbb{E}(\mu)^2 + \gamma c (2-c) \mathbb{E}(\mu)^2 < (1-\gamma) c^2 Var(\theta) + \gamma c (2-c) Var(\theta). \tag{9}$$

The right-hand side of this comparison is the joint payoff gain when θ is publicly observed. The maximum payoff gain when θ is public and $\gamma = \frac{1}{2}$ is equal to $Cov(\omega, \theta)$, implying that $c > 1$ is suboptimal. For $\gamma > \frac{1}{2}$, our discussion of the public inference case has shown that the right-hand side of (9) is increasing in $Var(\theta)$ for given $Cov(\omega, \theta)$, so again any $Cov(\omega, \theta), Var(\theta)$ such that $c > 1$ is suboptimal and dominated by an information structure satisfying $Var(\theta) = Cov(\omega, \theta)$.

Consider now information structures $Cov(\omega, \theta), Var(\theta)$ such that $c < 1$. The joint payoff gain in the limit equilibrium with countably infinitely many induced actions is

$$(1-\gamma) c^2 \mathbb{E}(\mu)^2 + \gamma c (2-c) \mathbb{E}(\mu)^2 = (1-\gamma) \frac{c}{2-c} Cov(\omega, \theta) + \gamma Cov(\omega, \theta) < Cov(\omega, \theta),$$

where the inequality follows from the fact that $\frac{c}{2-c} < 1$ for $c < 1$.

The gist of the argument is straightforward. The point is that for $\gamma > \frac{1}{2}$, the optimal information structure for public inferences puts more weight on sender relevant information. For given $Cov(\omega, \theta)$, information structures are intrinsically better for the sender the higher is $Var(\theta)$. Hence it is natural that any information structure satisfying $c > 1$ is very inefficient also under private inferences. For $c < 1$ and private inferences, the receiver reacts conservatively to the sender's advice. Hence $\mathbb{E}(\mu)^2$ attains at most a value of $\frac{Var(\theta)}{2-c}$, so strictly less than $Var(\theta)$. As a result, even though intrinsically more useful, information structures satisfying $c < 1$ lose all their appeal to the sender, because the sender gets a payoff gain of $\gamma Cov(\omega, \theta)$ for all such information structures, precisely also what he would gain for an information structure featuring $Var(\theta) = Cov(\omega, \theta)$. On top of this, the receiver gets a strictly smaller payoff gain from any information structure satisfying $c < 1$.

This extension shows that identical payoffs are not essential for our main insight. Two features are important for the result. Firstly, that the sender does not gain too much from information structures that are intrinsically more useful to him. Secondly, it is essential that the designer does not aim primarily at maximizing the receiver's expected utility.

To see the importance of the latter argument, take an extreme example where only the receiver's expected utility counts, $\gamma = 0$. Moreover, for simplicity assume that $\sigma_\eta^2 = \sigma_\omega^2 = \sigma^2$

and let $\rho \leq \frac{1}{2}$. Recall that the optimal information structure under public inferences is $Cov(\omega, \theta) = \sigma_{\omega\eta}$ and $Var(\theta) = \rho\sigma_{\omega\eta}$, implying that $c = \frac{\sigma_{\omega\eta}}{\rho\sigma_{\omega\eta}} = \frac{1}{\rho} \geq 2$. Suppose that the same information structure is also chosen under private inferences.

It is easy to show that for $c \geq 2$, the most informative communication equilibrium is a two partition equilibrium, inducing only two equilibrium actions, $x^+ = c \cdot \mu^+ = \frac{c}{\lambda}$ and $x^- = c \cdot \mu^- = -\frac{c}{\lambda}$. The receiver's expected utility gain from information acquisition is

$$c^2 \mathbb{E}(\mu)^2 = \frac{c^2}{\lambda^2} = \frac{1}{\rho}\frac{Cov(\omega, \theta)}{Var(\theta)}\frac{Var(\theta)}{2} = \frac{1}{\rho}\frac{1}{2}\sigma_{\omega\eta}.$$

By contrast, in the smooth communication outcome, where $Cov(\omega, \theta) = Var(\theta) = \sigma_{\omega\eta}$, the receiver's expected utility gain is

$$c^2 \mathbb{E}(\mu)^2 = Var(\theta) = \sigma_{\omega\eta}.$$

Hence the receiver prefers communication with only two messages and her preferred information structure to smooth communication under the balanced information structure, whenever $\rho \leq \frac{1}{2}$, that is, whenever the binary communication equilibrium is the most informative equilibrium.

This example shows that for pronounced conflicts and a sufficiently strong emphasis on the receiver's expected utility, smooth communication is not an equilibrium outcome; the organization prefers to sacrifice detailed communication in favor of eliminating residual risks for the receiver. Clearly, the example does not claim to characterize the optimum for the organization. This is a complex task that we leave to future work.

6 Authority in organizations

Several lines of thought in our theory appear already, without a formal model, in March and Simon (1958). In their description of problem-solving, the authors note that: "The design of the search process is itself often an object of rational decision." (p.140). In their discussion of communication processes inside an organization, the authors coin the term *uncertainty absorption* and describe its consequences as follows:

> "Uncertainty absorption takes place when inferences are drawn from a body of evidence and the inferences, instead of the evidence itself, are then communicated. [...] Both the amount and the *locus of uncertainty absorption affect the influence structure of the organization.* Because of this, uncertainty absorption is frequently used, consciously or unconsciously, as a technique for acquiring and exercising power. [...] Whatever may be the position in the organization holding the formal authority to legitimize the decision, to a considerable extent the effective discretion is exercised at the points of uncertainty absorption."
> (March and Simon (1958), pp 165–167, emphasis in original)

In our model, uncertainty absorption corresponds to the sender drawing a unidimensional inference - a conditional expectation - from multiple signals.[23] And indeed, although the receiver is formally legitimized to make the decision, the effective discretion is in fact exercised by the sender. This goes so far that communication and delegation become outcome equivalent. Given optimal information, allocating formal authority to the informed sender or bringing the information to the receiver are two ways to reach exactly the same outcome.

Our opening lines are inspired by the picture of organizations drawn by Cyert and March (1963), in particular their insightful discussion of communication and information acquisition (Chapter 4). The ideas that information needs to be acquired, that the search for information is endogenous, and that the communication system influences the information that is acquired, all appear in their work. Our contribution is to offer a formal model that puts these elements together and hopefully advances our understanding of them. Our main result is that decisions can be steered indirectly by choosing what issues to look into and how deeply to probe into them. While it may be surprising how well this works in principle, it seems obvious that it does work in practice. Indeed, Cyert et al. (1958) offer case study evidence consistent with our theory. The authors followed a medium-large manufacturing concern in the 1950s in the process of installing an electronic data-processing system. It was quickly decided that an outside consulting firm was needed. An offer was obtained

[23] In this sense, our contribution relates to the literature on multidimensional cheap talk. See Battaglini (2002), Meyer et al. (2013), Chakraborty and Harbaugh (2007), Chakraborty and Harbaugh (2010), and Levy and Razin (2007). These papers are not concerned with the impact of the quality of multidimensional information on communication.

from a consulting firm named Alpha in the study. There was an important person in the manufacturing concern, named the controller. After Alpha had made its offer, the controller decided that a competing offer should be requested from another firm; he selected a firm Beta out of a list of candidates that had been prepared beforehand. Beta delivered its offer. A memorandum was prepared at the request of the controller that listed the criteria that should be looked at to compare the offers and reach a decision. The final staff memorandum on the decision clearly recommended to hire Beta, a recommendation that the controller accepted. The controller is cited with the words: "I asked the boys to set down the pros and cons. The decision was Beta. It was entirely their decision."(Cyert et al. (1958), p.332)

Of course, we will never know why the boys favored Beta over Alpha; it could be that they wanted to please the controller or that Beta made the better offer. However, there is no account of explicit manipulation in the study. The point is that the controller can steer the decision indirectly to the point that it doesn't really matter who takes the decision. Our paper shows that this is precisely how a benevolent controller should act.

We are not the first to take up Simon's concept of authority. Aghion and Tirole (1997) distinguish formal from real authority. The allocation of formal authority has important effects on initiative and participation when there are private costs of information acquisition. In contrast, we abstract from such costs and information is acquired by the organization itself. On top of this, our concept of real authority is different, allowing the receiver to amend proposals as in Crawford and Sobel (1982). We follow their seminal approach to strategic information transmission between a sender and a receiver with important differences in assumptions and results. In Crawford and Sobel (1982) the sender's information is exogenously given and the sender wishes to induce an action that exceeds the ideal action of the receiver in each state of the world by some constant. In contrast, information is endogenous here and the sender's bias relative to the receiver depends on the information the sender gets to see. As a result, honest communication about sender-optimal policies is never an equilibrium in the Crawford-Sobel model while it is precisely a feature of the optimum in our model. Our assumptions have important effects on the delegation versus communication trade-off. Dessein (2002) studies the allocation of formal authority in the Crawford-Sobel model and shows that delegating decision rights to the informed sender is always better than communicating whenever meaningful communication is possible at all. In contrast, if information influences the magnitude and direction of biases, which both depend on the realized state of the world, and the organization can adapt to the situation along the informational margin, then delegation and communication become perfect substitutes.

Alonso et al. (2008) study the allocation of formal authority in an organization where two divisions interact with a headquarters. Both divisions have some information and need to make choices, preferably in a coordinated way. The organization can choose between vertical communication where all information flows upwards to a headquarters or horizontal communication where one division communicates with the other and the latter is in charge of decision making for both divisions. Depending on the relative importance of coordinating actions and of adapting choices to local conditions either one or the other form of communication is optimal. We study the same organization but in a quite different situation, where the form of the organization is exogenously given and information instead is endogenous. Allowing headquarters (the designer) to choose the information that enters the organization makes different allocations of formal authority perfect substitutes in our model. Obviously, as should be emphasized, if information cannot be chosen perfectly optimal - perhaps due to mistakes in the process of information acquisition - then, the allocation of authority remains a key choice variable.

7 Conclusions

A designer influences a sender-receiver game by choice of the information structure that the sender gets to observe. Provided that a priori known conflicts are eliminated, the designer can choose the information structure so as to enable effective communication between the sender and the receiver. We derive this outcome as the unique optimum from the perspective of joint surplus maximization. Designing the sender's information appropriately substitutes for other adjustments in the organization. Indeed, if the information is chosen optimally, then the allocation of authority becomes irrelevant: communication and delegation are outcome equivalent.

Many other interesting questions can be pursued in our environment. In companion work (Deimen and Szalay (2016)) we study a delegated expertise problem where the sender acquires information. Other scenarios are easy to imagine. Costs of information acquisition are a natural thing to look into now that the value of information is understood. We are pursuing some of these questions in ongoing work and leave many more for future work.

A Appendix

Lemma A.1 *Let Y follow an elliptical distribution, $Y \sim EC_n(\mu, \Sigma, \phi)$. Further let*

$$Y = (Y_1, Y_2), \quad \mu = (\mu_1, \mu_2), \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where the dimensions of Y_1, μ_1 and Σ_{11} are m, m, and $m \times m$.

i) The elliptical distribution is symmetric about μ.

ii) Linear combinations of elliptically distributed random variables are again elliptical.

iii) The conditional distribution of $(Y_1|Y_2 = y_2)$ is elliptical, with conditional mean vector

$$\mathbb{E}[Y_1|Y_2 = y_2] = \mu_1 + (y_2 - \mu_2)\Sigma_{22}^{-1}\Sigma_{21} \tag{A1}$$

and conditional covariance matrix satisfying

$$\Sigma^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}. \tag{A2}$$

Proof of Lemma A.1. i) by definition, ii) Fang et al. (1990) Theorem 2.16, iii) Fang et al. (1990) Theorem 2.18. ■

Proof of Lemma 1. Let $u \equiv u^R = u^S$ and $z = \omega, \eta$. Consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x - z) f(z|\, s_\omega, s_\eta)\, dz,$$

where $f(z|\, s_\omega, s_\eta)$ is the conditional density of $z = \omega, \eta$ given the signals. Since the utility depends only on the distance between x and z we have $u'(x - z) > 0$ for $z < x$, $u'(x - z) = 0$ for $x = z$, and $u'(x - z) < 0$ for $z > x$.

Consider the candidate solution $x^* = \mu_z \equiv \mathbb{E}[z|\, s_\omega, s_\eta]$. The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - z) f(z|\, s_\omega, s_\eta)\, dz = \int_{-\infty}^{\infty} u'(\mu_z - z) f(z|\, s_\omega, s_\eta)\, dz = 0.$$

Consider two points $z_1 = \mu_z - \Delta$ and $z_2 = \mu_z + \Delta$ for arbitrary $\Delta > 0$. By symmetry of u around its bliss point and symmetry of the distribution around μ_z, we have

$$u'(\Delta) f(\mu_z - \Delta | s_\omega, s_\eta) = -u'(-\Delta) f(\mu_z + \Delta | s_\omega, s_\eta).$$

Since this holds point-wise for each Δ, it also holds if we integrate over Δ. Thus, the first-order condition is satisfied at $x^* = \mu_z$. By concavity of u in x, only one value of x satisfies the first-order condition.

Applying equation $(A1)$, the conditional expectations are

$$\mathbb{E}[\eta | s_\omega, s_\eta] = \alpha^S s_\omega + \beta^S s_\eta \tag{A3}$$

and

$$\mathbb{E}[\omega | s_\omega, s_\eta] = \alpha^R s_\omega + \beta^R s_\eta, \tag{A4}$$

where the weights in the sender's ideal choice are

$$\alpha^S = \sigma^2_{\varepsilon_\eta} \frac{\rho \sigma_\omega \sigma_\eta}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho \sigma_\omega \sigma_\eta)^2}$$

and

$$\beta^S = \sigma^2_\eta \frac{\sigma^2_{\varepsilon_\omega} - \sigma^2_\omega \rho^2 + \sigma^2_\omega}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho \sigma_\omega \sigma_\eta)^2}$$

and the weights in the receiver's ideal choice are

$$\alpha^R = \sigma^2_\omega \frac{\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta \rho^2}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - (\rho \sigma_\omega \sigma_\eta)^2}$$

and

$$\beta^R = \sigma^2_{\varepsilon_\omega} \frac{\sigma_\eta \sigma_\omega \rho}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - (\rho \sigma_\omega \sigma_\eta)^2}.$$

First, suppose $\sigma^2_{\varepsilon_\eta}$ and $\sigma^2_{\varepsilon_\omega}$ are both positive and finite. Equations $(A3)$ and $(A4)$ are identical for all s_ω and s_η if and only if

$$\sigma^2_{\varepsilon_\eta} \rho \sigma_\omega \sigma_\eta = \sigma^2_\omega \left(\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta \rho^2\right)$$

and

$$\sigma_\eta^2 \left(\sigma_{\varepsilon_\omega}^2 - \sigma_\omega^2 \rho^2 + \sigma_\omega^2\right) = \sigma_\eta \sigma_\omega \rho \sigma_{\varepsilon_\omega}^2.$$

This requires that

$$\sigma_\eta^2 \left(1 - \rho^2\right) = \left(\frac{\rho \sigma_\eta}{\sigma_\omega} - 1\right) \sigma_{\varepsilon_\eta}^2$$

and

$$\sigma_\omega^2 \left(1 - \rho^2\right) = \left(\frac{\sigma_\omega \rho}{\sigma_\eta} - 1\right) \sigma_{\varepsilon_\omega}^2.$$

A necessary and sufficient condition for these two conditions to hold simultaneously is $\sigma_{\omega\eta} = \sigma_\eta^2 = \sigma_\omega^2$.

Consider now the limiting cases where one of the variances goes out of bounds. Applying l'Hôpital's rule to $(A3)$ and $(A4)$, we get in the limit as $\sigma_{\varepsilon_\eta}^2 \to \infty$

$$\mathbb{E}\left[\omega | s_\omega\right] = \frac{\sigma_\omega^2}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega \qquad \text{and} \qquad \mathbb{E}\left[\eta | s_\omega\right] = \frac{\rho_{\omega\eta} \sigma_\omega \sigma_\eta}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega,$$

so that

$$\mathbb{E}\left[\omega | s_\omega\right] \equiv \mathbb{E}\left[\eta | s_\omega\right] \qquad \Leftrightarrow \qquad \rho \sigma_\eta = \sigma_\omega.$$

Likewise, for the case where $\sigma_{\varepsilon_\omega}^2 \to \infty$, we get

$$\mathbb{E}\left[\omega | s_\eta\right] = \frac{\rho \sigma_\omega \sigma_\eta}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta \qquad \text{and} \qquad \mathbb{E}\left[\eta | s_\eta\right] = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta,$$

so

$$\mathbb{E}\left[\omega | s_\eta\right] \equiv \mathbb{E}\left[\eta | s_\eta\right] \qquad \Leftrightarrow \qquad \rho \sigma_\omega = \sigma_\eta.$$

■

Proof of Lemma 2. The proof is structured as follows. In Part i) we show that the sender's preferences over actions depend only on the distance of these actions to the sender's posterior expected mean. In Part ii) we argue that any equilibrium can also be characterized as one where sender types are restricted to play strategies that depend on θ only.

Part i) Let $u \equiv u^R = u^S$. Recall from Lemmas 1 and A.1 that $\theta = \mathbb{E}\left[\eta | s_\omega, s_\eta\right]$ and that the conditional distribution of η given s_ω, s_η is symmetric about θ. We first show that the sender's preferences over messages depend only on the distance between induced actions and θ. Let $x' - \mathbb{E}[\eta | s_\omega, s_\eta] = \mathbb{E}[\eta | s_\omega, s_\eta] - x'' \equiv z > 0$, then

$$\int u(x' - \eta) f(\eta | s_\omega, s_\eta) d\eta = \int u(z - (\eta - \mathbb{E}[\eta | s_\omega, s_\eta])) f(\eta | s_\omega, s_\eta) d\eta.$$

The random variable $\hat{\eta} \equiv \eta - \mathbb{E}[\eta | s_\omega, s_\eta]$ has mean zero and follows a symmetric distribution. Let $\hat{f}(\hat{\eta} | s_\omega, s_\eta)$ denote the standardized distribution (with mean zero). Then, we have

$$f(\eta | s_\omega, s_\eta) = \hat{f}(\eta - \mathbb{E}[\eta | s_\omega, s_\eta] | s_\omega, s_\eta) = \hat{f}(\hat{\eta} | s_\omega, s_\eta).$$

Take two realizations $\hat{\eta}'$ and $\hat{\eta}'' = -\hat{\eta}'$ of $\hat{\eta}$. By construction, we have $|z - \hat{\eta}'| = |-z - \hat{\eta}''|$ and hence by symmetry of u around 0, $u(z - \hat{\eta}') = u(-z - \hat{\eta}'')$. Symmetry of the distribution around zero is equivalent to $\hat{f}(\hat{\eta}' | s_\omega, s_\eta) = \hat{f}(\hat{\eta}'' | s_\omega, s_\eta)$. Therefore, for all $\hat{\eta}'$ we have $u(z - \hat{\eta}') \hat{f}(\hat{\eta}' | s_\omega, s_\eta) = u(\hat{\eta}' - z) \hat{f}(\hat{\eta}' | s_\omega, s_\eta)$, implying that

$$\int u(z - \hat{\eta}) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta} = \int u(\hat{\eta} - z) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta}.$$

By symmetry of the distribution, $\hat{\eta}$ and $-\hat{\eta}$ follow the exact same distribution, and we can write

$$\int u(\hat{\eta} - z) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta} = \int u(-z - \hat{\eta}) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta}.$$

Hence,

$$\begin{aligned}\int u(z - \hat{\eta}) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta} &= \int u(-z - \hat{\eta}) \hat{f}(\hat{\eta} | s_\omega, s_\eta) d\hat{\eta} \\ &= \int u(-z - (\eta - \mathbb{E}[\eta | s_\omega, s_\eta])) f(\eta | s_\omega, s_\eta) d\eta \\ &= \int u(x'' - \eta) f(\eta | s_\omega, s_\eta) d\eta,\end{aligned}$$

that is, the sender is indifferent between actions that are equidistant from θ, $|\theta - x'| = |x'' - \theta|$. By concavity of the sender's payoff, the sender prefers action x' over x'' if and only if x' is closer to θ.

Part ii) From Part i) we know that every set of types $\mathbf{s}$ with $\alpha^S s_\omega + \beta^S s_\eta$ =constant induces at most two actions in equilibrium.

Let $\underline{x}(\theta)$ denote the lowest action that type θ induces and let $\overline{x}(\theta)$ denote the highest action that type θ induces. Suppose that the set of sender types with signal realizations $\alpha^S s_\omega + \beta^S s_\eta = \hat{\theta}$ is divided into two subsets, S_1 and S_2, such that types in S_1 induce action $\underline{x}\left(\hat{\theta}\right)$ and types in S_2 induce action $\overline{x}\left(\hat{\theta}\right)$. Then, it must be that these senders are indifferent between the induced actions, $\left|\hat{\theta} - \underline{x}\left(\hat{\theta}\right)\right| = \left|\overline{x}\left(\hat{\theta}\right) - \hat{\theta}\right|$. Thus, $\overline{x}\left(\hat{\theta}\right) > \hat{\theta} > \underline{x}\left(\hat{\theta}\right)$. Let $\overline{x}\left(\hat{\theta}\right) - \hat{\theta} \equiv \hat{\varepsilon} > 0$.

It follows immediately that no type can induce any action $x \in \left(\hat{\theta} - \hat{\varepsilon}, \hat{\theta} + \hat{\varepsilon}\right)$. If such an action could be induced, then types with $\theta = \hat{\theta}$ could deviate and induce an action that is closer to their most preferred choice, $x = \hat{\theta}$. Likewise, all types $\theta \in \left[\hat{\theta}, \hat{\theta} + \hat{\varepsilon}\right]$ must also induce action $\overline{x}\left(\hat{\theta}\right)$. If we had $\underline{x}(\theta) > \hat{\theta} + \hat{\varepsilon}$ instead for some types with values of θ within this set, then these type would prefer to induce action $\overline{x}\left(\hat{\theta}\right)$ rather than $\underline{x}(\theta)$. A similar argument can be given for types $\theta \in \left[\hat{\theta} - \hat{\varepsilon}, \hat{\theta}\right]$ and $\overline{x}(\theta)$. It follows that types $\theta \in \left(\hat{\theta}, \hat{\theta} + \hat{\varepsilon}\right]$ induce a unique action $x(\theta) = \overline{x}\left(\hat{\theta}\right)$ and types $\theta \in \left[\hat{\theta} - \hat{\varepsilon}, \hat{\theta}\right)$ induce a unique action $x(\theta) = \underline{x}\left(\hat{\theta}\right)$. For both actions, there is a set of types who have positive mass that induce the action.

There are now two cases to distinguish: i) $\overline{x}\left(\hat{\theta}\right)$ is the highest action induced in equilibrium or ii) there is θ^* denoted as the smallest θ such that some types with $\theta = \theta^*$ induce a receiver action $\overline{x}(\theta^*)$ distinct from $\overline{x}\left(\hat{\theta}\right)$. In case i), it is easy to see that the set of types that induce two actions have zero mass and are pooled with a set of types of positive mass. In case ii), indifference of types with $\theta = \theta^*$ requires by construction that $\overline{x}\left(\hat{\theta}\right) = \underline{x}(\theta^*)$. Moreover, we must have $\overline{x}\left(\hat{\theta}\right) < \theta^* < \overline{x}(\theta^*)$ and $\left|\theta^* - \overline{x}\left(\hat{\theta}\right)\right| = \left|\overline{x}(\theta^*) - \theta^*\right|$. Again, for each action there is a set of types with positive mass that induce only this action. Moreover, a set of types with zero mass pools with them. Finally, note that the situation for types with $\theta = \theta^*$ corresponds exactly to the one we took as our starting point, so exactly the same arguments that have been applied to type $\theta = \hat{\theta}$ can be applied here.

Consider now the receiver's strategy. The receiver's action $\overline{x}\left(\hat{\theta}\right)$ must be a best reply to the set of types that induce the action. The set of sender types who induce action $\overline{x}\left(\hat{\theta}\right)$ is given by the set of signal realizations such that $\alpha^S s_\omega + \beta^S s_\eta = \theta \in \left(\hat{\theta}, \theta^*\right)$ and subsets of sender types with $\alpha^S s_\omega + \beta^S s_\eta = \theta \in \left\{\hat{\theta}, \theta^*\right\}$. Note that $\Pr\left[\theta \in \left(\hat{\theta}, \theta^*\right)\right] > 0$ while

$\Pr\left[\theta \in \left\{\hat{\theta}, \theta^*\right\}\right] = 0$. Let all these types send some message $\hat{m}$ to induce action $\overline{x}\left(\hat{\theta}\right)$. The conditional distribution of ω given $\hat{m}$ does not depend on the strategies of types $\theta \in \left\{\hat{\theta}, \theta^*\right\}$, since these types are pooled with a set of positive measure that induce only one action. Hence, the receiver's optimal action given message $\hat{m}$ does not depend on the strategies of these measure zero types either.

It follows that every equilibrium is essentially equivalent to one where senders are restricted to play strategies based on θ only. Essentially means, that the receiver's equilibrium actions, the receiver's beliefs, and expected payoffs are exactly the same; sender strategies are the same with probability one. ■

Proof of Lemma 3. Letting $a \equiv \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}$ and $b \equiv \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}$ we can rewrite $Cov(\omega, \theta)$ and $Var(\theta)$ as

$$Cov(\omega, \theta) = \sigma_{\omega\eta} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2},$$

and

$$Var(\theta) = \sigma_\eta^2 \frac{a + b\rho^2 + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2}.$$

Consider first the set of feasible levels of $Cov(\omega, \theta) = C$. Note that for $a = 0$ or $b = 0$, the covariance is constant and equal to $\sigma_{\omega\eta}$. Moreover, the covariance is decreasing in a for given b and decreasing in b for given a. By l'Hôpital's rule, we have

$$\lim_{b\to\infty} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2} = \frac{1}{1 + a},$$

and

$$\lim_{a\to\infty} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2} = \frac{1}{1 + b}.$$

So, letting both a and b (in whatever order) go to infinity results in a covariance of zero. By continuity, any $C \in (0, \sigma_{\omega\eta}]$ can be generated by finite levels a, b. Including the case where no signal is observed at all, we can generate all $C \in [0, \sigma_{\omega\eta}]$.

Consider next the set of feasible $Var(\theta)$ for any given level $Cov(\omega, \theta) = C$. Distinguish two cases, i) $C = \sigma_{\omega\eta}$ and ii) $C \in [0, \sigma_{\omega\eta})$.

Case i) requires that $a = 0$ or $b = 0$ or both. If $b = 0$, then $\frac{a+b\rho^2+1-\rho^2}{(1+a)(1+b)-\rho^2} = 1$ and thus $Var(\theta) = \sigma_\eta^2$ for all a. If $a = 0$, then

$$Var(\theta) = \sigma_\eta^2 \frac{b\rho^2 + 1 - \rho^2}{(1 + b) - \rho^2}$$

is decreasing in b and attains value $Var(\theta) = \sigma_\eta^2$ for $b = 0$. Moreover,

$$\lim_{b\to\infty} \frac{b\rho^2 + 1 - \rho^2}{(1+b) - \rho^2} = \rho^2.$$

Hence, for $C = \sigma_{\omega\eta}$, $Var(\theta) \in \left[\rho^2\sigma_\eta^2, \sigma_\eta^2\right]$; the lower limit is included because we allow for the case where only one signal is observed.

Case ii) $C \in [0, \sigma_{\omega\eta})$ requires that $a > 0$ and $b > 0$. Let $\delta \equiv \frac{C}{\sigma_{\omega\eta}} \in [0,1)$. The combinations of a and b that generate C satisfy

$$\frac{a + b + 1 - \rho^2}{(1+a)(1+b) - \rho^2} = \delta.$$

Solving for a as a function of b, we obtain

$$a(b;\delta) = \frac{(1-\delta)(1+b-\rho^2)}{\delta b - (1-\delta)} = \frac{(1+b-\rho^2)}{\frac{\delta}{1-\delta}b - 1}.$$

The function $a(b;\delta)$ is decreasing in b and has the limit

$$\lim_{b\to\infty} \frac{1+b-\rho^2}{\frac{\delta}{1-\delta}b - 1} = \frac{1-\delta}{\delta}.$$

In the limit as $b \to \frac{1-\delta}{\delta}$, we obtain $a \to \infty$. Hence, C can be generated for $b > \frac{1-\delta}{\delta}$ and $a = \frac{\left(1+b-\rho^2\right)}{\frac{\delta}{1-\delta}b-1}$. Substituting for $\frac{\left(1+b-\rho^2\right)}{\frac{\delta}{1-\delta}b-1}$ into $Var(\theta)$, we obtain

$$Var(\theta; b, a(b;\delta), \delta) = \sigma_\eta^2 \frac{\frac{\left(1+b-\rho^2\right)}{\frac{\delta}{1-\delta}b-1} + b\rho^2 + 1 - \rho^2}{\left(1 + \frac{\left(1+b-\rho^2\right)}{\frac{\delta}{1-\delta}b-1}\right)(1+b) - \rho^2} = \sigma_\eta^2 \frac{b\delta\rho^2 + 1 - \rho^2}{1 + b - \rho^2}.$$

The derivative of this expression in b is $\frac{\left(\delta\rho^2-1\right)\left(1-\rho^2\right)}{\left(1+b-\rho^2\right)^2} < 0$, so $Var(\theta; b, a(b;\delta), \delta)$ is continuous and monotone decreasing in b. In the limit as b tends to infinity, we obtain

$$\lim_{b\to\infty} \sigma_\eta^2 \frac{b\delta\rho^2 + 1 - \rho^2}{1 + b - \rho^2} = \sigma_\eta^2 \delta\rho^2 = \sigma_\eta^2 \frac{C}{\sigma_{\omega\eta}}\rho^2 = \frac{\sigma_\eta}{\sigma_\omega}\rho C.$$

In the limit as $b \to \frac{1-\delta}{\delta}$, we obtain

$$\lim_{b\to\frac{1-\delta}{\delta}} \sigma_\eta^2 \frac{b\delta\rho^2 + 1 - \rho^2}{1 + b - \rho^2} = \sigma_\eta^2 \frac{\frac{1-\delta}{\delta}\delta\rho^2 + 1 - \rho^2}{1 + \frac{1-\delta}{\delta} - \rho^2} = \delta\sigma_\eta^2 = \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C.$$

Hence, we have shown that for any given $C \in [0, \sigma_{\omega\eta})$, $Var(\theta) \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho C, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C\right]$. We include the lower limit, because the case where $b \to \infty$ is equivalent to the case with one signal only. ■

Proof of Theorem 1. Let $u \equiv u^R = u^S$, $C \equiv Cov(\omega, \theta)$, and $V \equiv Var(\theta)$. We prove the theorem in two steps. In step i) we derive the standardized distributions. In step ii) we solve the maximization problem.

i) Let $f_{\omega\theta}(\omega, \theta) = \int f(\omega, \eta, \theta)\, d\eta$ and let $f_{\eta\theta}(\omega, \theta) = \int f(\omega, \eta, \theta)\, d\omega$ denote the marginal joint densities of ω, θ and η, θ. Consider first the expected utility of the sender.

Let $\tau \equiv \frac{C}{V}\theta - \eta$ and let $g(\cdot)$ denote the density of τ. The expected utility of the sender satisfies

$$\int\int u\left(\frac{C}{V}\theta - \eta\right) f_{\eta\theta}(\eta, \theta)\, d\eta d\theta = \int\int u(\tau) f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\tau d\theta$$
$$= \int u(\tau) \int f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\theta d\tau = \int u(\tau) g(\tau)\, d\tau = \int u(\sigma_\tau t)\, \kappa\phi(t)\, dt.$$

For the first equality, substitute τ and apply the switch of variables theorem. For the second, apply Fubini's theorem. For the third, note that $\Pr\left[\frac{C}{V}\theta - \eta \le \tau\right] = \Pr\left[\frac{C}{V}\theta - \tau \le \eta\right]$ and that by Leibniz's rule

$$g(\tau) = \frac{\partial}{\partial\tau}\Pr\left[\frac{C}{V}\theta - \eta \le \tau\right] = \frac{\partial}{\partial\tau}\int_{-\infty}^{\infty}\int_{\frac{C}{V}\theta-\tau}^{\infty} f_{\eta\theta}(\eta, \theta)\, d\eta d\theta = \int_{-\infty}^{\infty} f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\theta.$$

Since τ is a linear function of θ and η, we can use Fang et al. (1990) Theorem 2.16 to conclude that $g(\tau)$ is the density of an elliptical distribution that has the same characteristic generator, $\phi(\cdot)$, as f has. The variance of τ is

$$\begin{aligned}\sigma_\tau^2 &= \frac{C^2}{V^2}Var(\theta) - 2\frac{C}{V}Cov(\theta, \eta) + Var(\eta) \\ &= \frac{C^2}{V} - 2C + \sigma_\eta^2.\end{aligned}$$

Standardizing to $t = \frac{\tau}{\sigma_\tau}$, we transform to a spherical (standardized elliptical) distribution with density $\kappa\phi(\cdot)$.

To derive the receiver's expected utility, we let $\zeta \equiv \frac{C}{V}\theta - \omega$, let $h(\cdot)$ denote the density of ζ. Going through the exact same steps one finds that $h(\zeta) = \int f\left(\frac{C}{V}\theta - \zeta, \theta\right) d\theta$, again an elliptical density with the same characteristic generator. The variance of ζ is

$$\sigma_\zeta^2 = \sigma_\omega^2 - \frac{C^2}{V}.$$

Hence, with $z = \frac{\zeta}{\sigma_\zeta}$, we can write

$$\int\int u\left(\frac{C}{V}\theta - \omega\right) f_{\omega\theta}(\omega,\theta)\, d\omega d\theta = \int u\left(z\sigma_{\omega|\theta}\right)\kappa\phi(z)\, dz.$$

ii) An optimal information structure solves:

$$\max_{C,V} \int u\left(z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{\frac{1}{2}}\right)\kappa\phi(z)\, dz + \int u\left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\, dt.$$

We solve the problem by maximizing sequentially wrt C and V. For given C, the derivative wrt V is

$$\begin{aligned} &\frac{1}{2}\frac{C^2}{V^2}\int z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{\frac{1}{2}}\right)\kappa\phi(z)\, dz \\ &-\frac{1}{2}\frac{C^2}{V^2}\int \left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{-\frac{1}{2}} tu'\left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\, dt. \end{aligned} \tag{A6}$$

Recall that $\sigma_\eta^2 = \sigma_\omega^2$. First, suppose $V = C$. Then, the derivative wrt V satisfies

$$\begin{aligned} &\int \frac{1}{2} z\left(\sigma_\omega^2 - V\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - V\right)^{\frac{1}{2}}\right)\kappa\phi(z)\, dz \\ &-\int \frac{1}{2}\left(-V + \sigma_\eta^2\right)^{-\frac{1}{2}} tu'\left(\left(-V + \sigma_\eta^2\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\, dt \\ = \; & 0. \end{aligned}$$

Now suppose $V \neq C$. Note that both integrands in (A6) have the common representation

$$\int \frac{1}{a} ku'(ak)\kappa\phi(k)\, dk. \tag{A7}$$

Differentiating wrt a, we observe that (A7) is monotone decreasing in a,

$$-\frac{1}{a^3}\int aku'(ak)\kappa\phi(k)\, dk + \frac{1}{a^3}\int a^2k^2u''(ak)\kappa\phi(k)\, dk \le 0,$$

where the inequality follows from the curvature condition

$$q\frac{u''(q)}{u'(q)} = q\frac{\ell''(q)}{\ell'(q)} \geq 1. \tag{A8}$$

$V < C$ implies $\frac{C^2}{V} - 2C + \sigma_\eta^2 > \sigma_\omega^2 - \frac{C^2}{V}$. The curvature condition (A8) implies monotonicity and therefore

$$\begin{aligned} & \frac{1}{2}\frac{C^2}{V^2}\int z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{\frac{1}{2}}\right)\kappa\phi(z)\,dz \\ \geq\ & \frac{1}{2}\frac{C^2}{V^2}\int \left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{-\frac{1}{2}} tu'\left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\,dt. \end{aligned}$$

Hence the derivative is non-negative for $V < C$. By symmetry, the derivative is non-positive for $V > C$. These inequalities become strict for functions that satisfy the curvature condition (A8) with strict inequality. It follows that the problem is maximized in V for $V = C$.

The second step is now to maximize over C, given that $V = C$.

$$\max_C \int u\left(z\left(\sigma_\omega^2 - C\right)^{\frac{1}{2}}\right)\kappa\phi(z)\,dz + \int u\left(\left(\sigma_\eta^2 - C\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\,dt.$$

The derivative wrt C is given by

$$\begin{aligned} & -\int \frac{1}{2} z\left(\sigma_\omega^2 - V\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - V\right)^{\frac{1}{2}}\right)\kappa\phi(z)\,dz \\ & -\int \frac{1}{2}\left(\sigma_\eta^2 - C\right)^{-\frac{1}{2}} tu'\left(\left(\sigma_\eta^2 - C\right)^{\frac{1}{2}} t\right)\kappa\phi(t)\,dt \\ >\ & 0. \end{aligned}$$

The payoff is unambiguously increasing in C. The solution is thus $C = C^{\max}$. ■

Proof of Proposition 1. By Theorem 1, for quadratic loss functions all information structures satisfying $Cov(\omega,\theta) = \sigma_{\omega\eta}$ are optimal for θ public. By Theorem 2, smooth communication is an equilibrium if and only if $Cov(\omega,\theta) = Var(\theta)$. By Lemma 3, the candidate solution $Var(\theta)^* = Cov(\omega,\theta)^* = \sigma_{\omega\eta}$ is feasible if $\frac{Cov(\omega,\theta)^*}{Var(\theta)^*} = 1 \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}\right]$. This is guaranteed by the assumption $\min\left\{\sigma_\omega^2, \sigma_\eta^2\right\} \geq \sigma_{\omega\eta}$. ■

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